The CAPM

Class 11
Financial Management, 15.414
Today

The CAPM

- Measuring risk
- Systematic vs. diversifiable risk
- The trade-off between risk and return

Reading

- Brealey and Myers, Chapter 8.2 – 8.5
Review

Diversification

- Diversification reduces risk, as long as stocks aren’t perfectly correlated with each other.

- Portfolio variance depends primarily on the covariances among stocks, not the individual variances. Risk common to all firms cannot be diversified away.

- Investors should try to hold portfolios that maximize expected return for a given level of risk. The tangency portfolio is the best portfolio.
Diversification

If correlation = 1.0
If correlation = 0.4
If correlation = 0.0

Std dev of portfolio vs. Number of stocks

If correlation = 1.0
If correlation = 0.4
If correlation = 0.0

Number of stocks

If correlation = 1.0
If correlation = 0.4
If correlation = 0.0
Optimal portfolios

Mean

0.0% 2.0% 4.0% 6.0% 8.0% 10.0% 12.0% 14.0% 16.0%

Std dev

Riskfree asset

Tangency portfolio

GM

IBM

Motorola

Efficient frontier
The CAPM

Capital Asset Pricing Model

➢ Stock prices are affected by firm-specific and marketwide risks. Investors care only about risk that is non-diversifiable.

➢ A stock’s non-diversifiable risk is measured by beta, the slope when the stock is regressed on the market:

\[ R_i = \alpha + \beta R_M + \varepsilon \]

➢ Expected, or required, returns are a linear function of betas:

\[ E[R_i] = r_f + \beta_i E[R_M - r_f] \]

Market risk premium

For example, a stock with \( \beta = 2 \) is twice as risky as the market, so investors require twice the risk premium.
CAPM: Security Market Line

Slope = \( E[R_M] - r_f \)

\( \beta = 1.5 \)

\( \beta = 0.5 \)

\( \beta = 0 \)

Market portfolio (\( \beta = 1 \))
Beta

Regression slope

➤ **How sensitive is the stock to overall market movements?**
How much does the stock go up or down when other stocks go up or down?

➤ \( R_i = \alpha + \beta \ R_M + \epsilon \)

\( \epsilon \) = firm-specific return
('diversifiable,' 'idiosyncratic,' or 'unsystematic' risk)

\( \beta \) = sensitivity to market returns
('systematic,' 'non-diversifiable,' or 'macroeconomic' risk)

\( R^2 \) = explained variance
(fraction of variance explained by market returns)
Regressions in Excel
Gillette vs. Total U.S. market return

Monthly returns

\[ \beta = 0.81 \]

\[ R^2 = 0.19 \]
NASDAQ vs. Total U.S. market return

Monthly returns
$\beta = 1.57$
$R^2 = 0.77$
# Betas, 1960 – 2001

## Size-sorted portfolios

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>β</td>
<td>$R^2$</td>
<td>β</td>
<td>$R^2$</td>
</tr>
<tr>
<td>Smallest</td>
<td>1.58</td>
<td>0.60</td>
<td>Smallest</td>
<td>1.27</td>
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<tr>
<td>2</td>
<td>1.45</td>
<td>0.76</td>
<td>2</td>
<td>1.25</td>
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<tr>
<td>3</td>
<td>1.45</td>
<td>0.81</td>
<td>3</td>
<td>1.26</td>
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<td>1.36</td>
<td>0.84</td>
<td>4</td>
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<td>0.92</td>
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<td>1.09</td>
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<td>8</td>
<td>1.16</td>
<td>0.95</td>
<td>8</td>
<td>1.04</td>
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<tr>
<td>9</td>
<td>1.05</td>
<td>0.96</td>
<td>9</td>
<td>1.02</td>
</tr>
<tr>
<td>Largest</td>
<td>0.92</td>
<td>0.97</td>
<td>Largest</td>
<td>0.96</td>
</tr>
</tbody>
</table>
Key insight

For a diversified investor, beta measures a stock’s contribution to portfolio risk. Beta, not variance, is the appropriate measure of risk.

The required return on a stock equals:

\[ E[R_i] = r_f + \beta_i E[R_M - r_f] \]
Security Market Line

\[ \text{Slope} = E[R_M] - r_f \]

\[ \beta = 0 \]

\[ \beta = 0.5 \]

\[ \beta = 1.5 \]

Market portfolio (\( \beta = 1 \))
Example 1

Using monthly returns from 1990 – 2001, you estimate that Microsoft has a beta of 1.49 (std err = 0.18) and Gillette has a beta of 0.81 (std err = 0.14). If these estimates are a reliable guide for their risks going forward, what rate of return is required for an investment in each stock?

\[ E[R_i] = r_f + \beta_i (E[R_M] - r_f) \]

Tbill rate = 1.0%; market risk premium is around 4 – 6%.

Expected returns

- **Gillette:** \( E[R_{GS}] = 0.01 + (0.81 \times 0.06) = 5.86\% \)
- **Microsoft:** \( E[R_{MSFT}] = 0.01 + (1.49 \times 0.06) = 9.94\% \)
Example 2

Over the past 40 years, the smallest decile of firms had an average monthly return of 1.33% and a beta of 1.40. The largest decile of firms had an average return of 0.90% and a beta of 0.94. Over the same time period, the riskfree rate averaged 0.43% and the market risk premium was 0.49%. Are the average returns consistent with the CAPM?

\[ E[R_{i}] = r_{f} + \beta_{i} (E[R_{M}] - r_{f}) \]

Tbill rate = 0.43%; market risk premium is 0.49%.

How far are average returns from the CAPM security market line?
## Size portfolios, 1960 – 2001

### Average returns vs. CAPM

<table>
<thead>
<tr>
<th>Decile</th>
<th>Avg return</th>
<th>$\beta$</th>
<th>$r_f + \beta_i E[R_M - r_f]$</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smallest</td>
<td>1.33</td>
<td>1.40</td>
<td>1.15</td>
<td>0.19</td>
</tr>
<tr>
<td>2</td>
<td>1.06</td>
<td>1.33</td>
<td>1.11</td>
<td>-0.06</td>
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<td>3</td>
<td>1.13</td>
<td>1.34</td>
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<td>0.01</td>
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<td>1.14</td>
<td>1.28</td>
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<td>0.05</td>
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<td>0.07</td>
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<td>1.10</td>
<td>1.19</td>
<td>1.04</td>
<td>0.06</td>
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<td>7</td>
<td>1.04</td>
<td>1.14</td>
<td>1.02</td>
<td>0.02</td>
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<tr>
<td>8</td>
<td>1.10</td>
<td>1.09</td>
<td>0.99</td>
<td>0.11</td>
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<tr>
<td>9</td>
<td>1.00</td>
<td>1.03</td>
<td>0.97</td>
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<td>0.93</td>
<td>-0.03</td>
</tr>
</tbody>
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Difference = Avg. return – CAPM prediction
Size portfolios, 1960 – 2001
Example 3

You are choosing between two mutual funds. Over the past 10 years, BlindLuck Value Fund had an average return of 12.8% and a β of 0.9. EasyMoney Growth Fund had a return of 17.9% and a β of 1.3. The market’s average return over the same period was 14% and the Tbill rate was 5%.

Which fund is better?

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Avg return</th>
<th>β</th>
<th>CAPM r_f + β_i E[R_M − r_f]</th>
<th>Dif</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>14.0%</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BlindLuck</td>
<td>12.8</td>
<td>0.9</td>
<td>13.1</td>
<td>-0.30</td>
</tr>
<tr>
<td>EasyMoney</td>
<td>17.9</td>
<td>1.3</td>
<td>16.7</td>
<td>1.20</td>
</tr>
</tbody>
</table>

[‘Dif’ is referred to as the fund’s ‘alpha’]
Example 3

![Graph showing the relationship between average return and beta. The graph has two lines: one representing the market and another for easy money. The market line intersects the easy money line at a beta of 1.0. Points labeled 'Market', 'Blind Luck', and 'Easy Money' are marked on the graph.](image-url)
CAPM

Applications

➤ Measures and quantifies ‘risk’
   One stock or project is riskier than another stock or project if it has a higher $\beta$.

➤ Valuation
   The CAPM provides a way to estimate the firm’s cost of capital (risk-adjusted discount rate).*

➤ Evaluating a stock or mutual fund’s risk-adjusted performance
   The CAPM provides a benchmark.

* Graham and Harvey (2000) survey CFOs; 74% of firms use the CAPM to estimate the cost of capital.
Observation 1

Portfolios

A portfolio’s beta is a weighted average of the betas of the individual stocks.

Stocks 1, …, N
Portfolio return: \( R_P = w_1 R_1 + w_2 R_2 + \ldots + w_N R_N \)

Individual stocks

\[
\begin{align*}
R_1 &= \alpha_1 + \beta_1 R_M + \varepsilon_1 \\
R_2 &= \alpha_2 + \beta_2 R_M + \varepsilon_2 \\
&\vdots \\
R_N &= \alpha_N + \beta_N R_M + \varepsilon_N
\end{align*}
\]

Portfolio

\[
R_P = \alpha_P + \beta_P R_M + \varepsilon_P
\]

What happens to the residual variance when more stocks are added?

avg of \( \beta_1, \ldots, \beta_N \)
Observation 1

Example

Two groups of stocks

Group 1: $\beta = 0.5$
Group 2: $\beta = 1.5$

All stocks have a standard deviation of 40%. The market portfolio has standard deviation of 20%.

How does portfolio beta and residual risk change as the portfolio gets more and more stocks?
Hypothetical portfolios vs. market portfolio

\[ \beta = 0.5, \ N = 10 \]

\[ \beta = 0.5, \ N = 50 \]

\[ \beta = 1.5, \ N = 10 \]

\[ \beta = 1.5, \ N = 50 \]
Diversification

Group 2: $\beta = 1.5$

Group 1: $\beta = 0.5$
Observation 2

Total variance vs. beta risk

Two assets can have the same total variance, but much different $\beta$’s. Investors should care only about systematic, beta, risk.

$$\text{var}(R_i) = \beta^2 \text{var}(R_M) + \text{var}(\varepsilon_i)$$

Which stock is riskier?

**Stock 1:** $\text{std}(R_1) = 0.40$, $\beta = 0.5$

**Stock 2:** $\text{std}(R_2) = 0.40$, $\beta = 1.5$
Observation 3

Assets can have negative risk!

A stock’s $\beta$ is less than 0 if the stock is negatively correlated with the market portfolio. If the market goes down, it goes up.

Such a stock contributes negatively to portfolio risk. The stock is better than riskfree!

Examples
Various derivative securities; return from a short sale of stock
Observation 4

Tangency portfolio

The CAPM implies that the market portfolio should be the tangency portfolio.
The market portfolio will have the highest risk-return trade-off (or Sharpe ratio) of any possible portfolio.

You cannot gain by stock-picking.
Competition among investors ensures that stock prices are efficient; the only way to earn a higher rate of return is to take more risk.

Portfolio advice
Buy an index fund (like Vanguard 500)