

# **OPTIMAL SHARING CONTRACTS**

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## ABSTRACT

This paper derives optimal sharing ratio under sharing contract (*mudharabah*). The contract then is compared to standard debt contract (*riba*) under symmetric and asymmetric information. It is found that, when bankruptcy costs are positive, and expected marginal gain is not less than marginal loss, aggregate expected profits from sharing exceed those of debt, under both symmetric and asymmetric information. This result holds despite that effort level is the same in both contracts.

Moreover, for a certain range of the opportunity cost, both the financier and the entrepreneur are better off signing a sharing contract instead of debt. Moving from sharing to debt, in contrast, cannot make both parties better off; only one party may be better off while the other must be worse off. Sharing therefore Pareto-dominates debt for the relevant range of the opportunity cost, while debt cannot dominate sharing.

The model is extended to include endogenous cost of bankruptcy and endogenous probability of success. These extensions confirm the above results regarding expected profits. Level of effort in debt, however, may be below or in excess of the efficient level, resulting in sub-optimal payoff.

The model is applied to different forms of sharing, including that of a manager-partner as well as partnerships or *sharikat 'inan*. Optimal sharing ratios in each case are derived.

Finally, the interaction between the financier and the entrepreneur is presented in a game-theoretic framework. The resulting game is shown to be a Prisoner's Dilemma game, where sharing is the cooperative outcome and debt is the defective outcome. The Prisoner's Dilemma provides many insights into the contracting problem, and how cooperation can evolve over time.

## 1. INTRODUCTION

A widely held perception among economists is that sharing arrangements are less efficient compared to first-best solutions. Stiglitz and Weiss (1981), for example, write: ‘In general, revenue sharing arrangements such as equity financing, or sharecropping are inefficient. Under those schemes the managers of a firm or the tenant will equate their marginal disutility of effort with their share of their marginal product rather than with their total marginal product. Therefore, too little effort will be forthcoming from agents’ (p. 407). The same problem arises in corporate management. Harris and Raviv (1991) write: ‘Conflicts between shareholders and managers arise because managers hold less than 100% of the residual claim. Consequently, they do not capture the entire gain from their profit enhancement activities, but they do bear the entire cost of these activities’ (p. 300).

To elaborate this argument, suppose return of an investment project is an increasing function of the entrepreneur’s effort  $e$ , such that  $R = R(e)$ ,  $R' > 0$ . Suppose that disutility of effort  $c(e)$  is also an increasing function in  $e$ , so that  $c' > 0$ . Then the entrepreneur will choose effort to maximize  $\pi = R(e) - c(e)$ . The first order condition is  $R' - c' = 0$ . Let that resulting optimal effort be  $e^*$ . Now suppose that the entrepreneur gets only a share  $\alpha$  of return. His first order condition now becomes  $\alpha R' - c' = 0$ . This equation cannot hold at  $e^*$ , since  $\alpha R'(e^*) < c'(e^*)$ . To rebalance the equation, therefore, effort has to be reduced to  $\hat{e}$ , such that  $\alpha R'(\hat{e}) - c'(\hat{e}) = 0$ . Since  $e^* > \hat{e}$ , sharing results in a lower level of effort than first-best solution. If the project is financed through a fixed-payment contract, like debt, then the entrepreneur has to pay a fixed amount  $r$ , which does not affect marginal conditions. Thus a fixed-payment contract is superior in terms of efficiency to a sharing contract.

This argument represents an essential ingredient in models of ‘optimal financial contracts.’ Another important ingredient, in presence of asymmetric information, is deterministic monitoring, where the financier monitors in some states but not others.

Combining a fixed-payment arrangement with deterministic monitoring, it trivially follows that debt becomes the optimal form of financing. Gale and Hellwig (1985) point to this result clearly: 'It is worth noting that the proof of the optimality of the standard debt contract follows from the definition of the problem. To prove the converse requires a number of non-trivial assumptions' (p. 648).

Despite the inefficiency argument, sharing is widely practiced. Venture capital is a prominent form of equity financing. Sharecropping is applied in developing as well as developed economies. Explanations for existence of sharing arrangements suggest a kind of tradeoff or balance of one form or another to compensate for reduction in efficiency. These include:

- Risk-sharing properties (Stiglitz, 1974, Newbery, 1977; Lang and Gordon, 1995; see also Rees, 1985, and Hart and Holmstrom, 1987);
- Transaction costs (Murrell, 1983; Allen and Lueck, 1993);
- Bargaining powers (Bell and Zusman, 1976; Reiersen, 2001);
- Double-sided moral hazard (Reid, 1973; Eswaran and Kotwal, 1985; Bhattacharyya and Lafontaine, 1995);
- Moral hazard over choice of project with limited liability (Jensen and Meckling, 1976; Bester and Hellwig, 1989; Basu, 1992; see also Sengupta, 1997); as well as joint moral hazard in choice of project and choice of effort (Ghatak and Pandey, 2000);
- Strategic interaction among principals (Ray, 1999);
- Intertemporal discounting (Roy and Serfes, 2001).

The objective of this paper is to show that under a widely observed form of uncertainty, sharing can be no less efficient than first-best solution, and thus no need for a form of tradeoff to account for its usefulness. With positive bankruptcy costs and sufficiently high probability of success, sharing Pareto-dominates debt in terms of expected profits, with and without informational asymmetry. Under more general conditions, it is shown that debt even ceases to attain efficient level of effort, while sharing preserves its first-best solution.

## 2. THE MODEL

We build a simple model to analyze sharing arrangements and compare it to debt. We start with an entrepreneur who is able to exert effort  $e > 0$ , to run a certain project in a single period. The project requires an investment of  $I = I(e)$ . Higher effort requires higher investment. For example, a farmer needs to spend more if he is to expend more effort for implanting the land. A pharmaceutical company needs to spend more if more research is needed to develop a new drug, and so on. In other words, effort drives investment.

Return of the project is also a function of effort, but it also depends on market conditions:  $R = R(e;d)$ , where  $d$  denotes state of demand. Demand could be high,  $h$ , or low,  $w$ . High demand results in return that exceeds investment, while low demand results in return below investment, i.e:  $R_h(e) > I(e) > R_w(e)$ , where  $R_d(e) = R(e;d)$ .

High demand (or success) occurs with probability  $q$ , while low demand (or failure) occurs with probability  $1 - q$ , whereby  $0 < q < 1$ .

We assume that both return and investment are concave functions in effort. Moreover, we assume that  $R'_h > I' > R'_w$ , and that  $R''_h < I'' < R''_w$ .

Define net return in case of high demand as  $R_g = R_h - I$ , and net return (in absolute value) when demand is low as  $R_l = I - R_w$ , where subscripts  $g$  and  $l$  denote gain and loss, respectively. Note that, from the above assumptions, both  $R_g$  and  $R_l$  are concave in effort.

Formulating both gain and loss as increasing functions of effort is consistent with a wide range of business activities in the real world. For example, a pharmaceutical firm could exert a lot of effort in developing a new drug. If demand for this new drug turns out to be 'high,' effort and expenses spent on its R&D would have a positive impact on returns. The same effort and expenses become a source of increasing loss if demand is low. Alternatively, a firm could launch a marketing campaign for a particular product. The stronger the campaign, the larger the gain in case of success, and the larger the loss in case of failure.

We have considered effort above as *managerial* effort, i.e. effort for managing a given project. We might as well view it as effort for *searching* for profitable projects.

The assumption is that there is a large number of projects with varying levels of expected profitability. Projects with low levels of profitability are easier to find than those with high levels. The higher the profitability, the harder it is to be spotted. Hence  $R_g$  arranges expected profitability of different projects as an increasing function of searching effort. Search, however, may or may not be successful. It succeeds with probability  $q$  and fails with probability  $1-q$ . This is in contrast to the managerial interpretation of effort, where  $q$  is probability that the given project is successful. If search is successful, the return is  $R_g$ . If not, the entrepreneur loses  $R_l$ , which represents loss in invested capital. The model therefore, allows for analyzing both project selection as well as managing a given project. For consistency, the managerial interpretation of effort will be adopted in this paper.

We assume that expected marginal gain is not less than marginal loss, so that  $qR'_g \geq R'_l$ . This assures the viability of the project. Further comments on this assumption will be presented later.

Disutility of effort is a convex function of effort,  $c(e) > 0$ , such that  $c' > 0$ ,  $c'' > 0$ .

Profit function for the entrepreneur is written as:

$$\pi_f(e) = qR_g(e) - (1-q)R_l(e) - c(e). \quad [1]$$

The subscript  $f$  denotes the case of self-financing. For the relevant range of effort, we assume that  $\pi_f(e) \geq 0$ . The entrepreneur therefore is risk neutral, and his decision problem is to choose effort to maximize  $\pi_f(e)$ .

We assume that  $\pi_f(e)$  reaches its unique maximum at  $e^f$ , such that  $\pi_f(e^f) > \pi_f(e)$ ,  $\forall e \neq e^f$ . Thus  $\pi'_f(e^f) = 0$  and  $\pi''_f(e^f) < 0$ .

## 2.1 FIRST ORDER CONDITION

If the entrepreneur has sufficient capital to start his business, he chooses effort to maximize  $\pi_f(e)$ . First order condition is:

$$qR'_g(e) - (1-q)R'_l(e) = c'(e). \quad [2]$$

Let the solution be  $e^f$ . Define  $\lambda(e^f)$  such that

$$\lambda(e^f) = q \left( 1 - \frac{(1-q)R'_l(e^f)}{qR'_g(e^f)} \right)$$

Then we can write [2] as:

$$\lambda(e^f)R'_g(e^f) = c'(e^f). \quad [3]$$

Now we show that the  $\lambda$  is a positive fraction:

PROPERTY 1:  $0 < \lambda(e^f) < 1$

*Proof:* Since  $c' > 0$ , first order condition [2] implies that, at  $e^f$ ,  $qR'_g > (1-q)R'_l$ .

As all terms are positive, then  $0 < (1-q)R'_l/qR'_g < 1$ . Since  $0 < q < 1$ , it follows that  $0 < \lambda < 1$ . Q.E.D.

This result shows that a self-financed entrepreneur gets a fraction of marginal return, while he bears full marginal cost. That is, *the inefficiency problem exists due to uncertainty with no sharing of any kind*. We argue that sharing arrangements could be designed to achieve the same level of effort of a self-financed entrepreneur, thus losing no efficiency compared to first-best solution.

## 2.2 UNCERTAINTY AND EFFICIENCY

That uncertainty as such leads to inefficiency is not new to economists. For example, a risk neutral producer would hire less labour under uncertainty than under certainty (McKenna, 1986, ch. 4). Muslim scholars were aware of this effect of uncertainty long time ago. In the Qur'ān, the Prophet (peace be upon him) is instructed to say: 'If I had known the unseen, I should have multiplied the good, and no evil should have touched me' (7:188). Interpreters comment: 'had I known the unseen, I should have prepared during the good season for the bad season, and should have known when prices are high and when they are low, so that I prepare for the former during the latter.' (Ibn Jareer, 1968, 9:143). So the fact that the unseen is unknown results in less good than if it were known, and that is the inefficiency resulting from uncertainty.

### 2.3 FORMS OF UNCERTAINTY

Many models applied in the contracting literature do not obtain inefficiency due to uncertainty. Two reasons can be cited:

1. The entrepreneur is assumed to observe the state of the world prior to starting the project. He therefore does not face uncertainty in the first place.

2. The entrepreneur faces only a special form of uncertainty. These models assume either a multiplicative or an additive form of uncertainty. That is, they assume return to be a function of a random error  $\tilde{\varepsilon}$  such that  $R(e) = \tilde{\varepsilon}F(e)$ , whereby  $E\tilde{\varepsilon} = 1$ , or  $R(e) = F(e) + \tilde{\varepsilon}$ , whereby  $E\tilde{\varepsilon} = 0$ . Since maximization is performed in terms of expected return, neither form of uncertainty affects marginal conditions, and thus optimal level of input is the same as in case of certainty. In the present model, however, uncertainty is represented through *state-dependent* return functions, and thus affects optimal level of inputs.

A legitimate question at this stage arises: what is the relationship between these forms of uncertainty, and which form is more relevant to reality? The following property attempts to provide an answer:

**PROPERTY 2** Both multiplicative and additive uncertainties are special cases of state-dependent uncertainty.

*Proof:* Let  $\tilde{\varepsilon}$  be a random variable such that:

$$\tilde{\varepsilon} = \begin{cases} \varepsilon_g \circ q \\ \varepsilon_l \circ 1 - q \end{cases} \quad \text{where } \varepsilon_g \text{ and } \varepsilon_l \text{ are positive constants, and '}\circ\text{' is a shorthand}$$

for 'with probability of.' The expected value of  $\tilde{\varepsilon}$  is  $\bar{\varepsilon} = q\varepsilon_g + (1 - q)\varepsilon_l$ .

1. For multiplicative uncertainty, let  $R_g = \varepsilon_g F$  and  $-R_l = -\varepsilon_l F$ , where  $F > 0$  is an increasing function of effort, and  $\varepsilon_g > \varepsilon_l$ . Hence we can write the expected profit function of a self-financed entrepreneur as  $\pi^f = \bar{\varepsilon}F - c$ . For suitable values of  $\varepsilon_g$  and  $\varepsilon_l$ , we can have  $\bar{\varepsilon} = 1$ , and thus  $\pi^f = F - c$ , which is identical to the profit function in case of certainty. Consequently, uncertainty has no impact on optimal effort.

2. For additive uncertainty, let realizations of  $\tilde{\varepsilon}$  be  $\varepsilon_g \circ q$  and  $-\varepsilon_l \circ 1 - q$ ,

where  $\varepsilon_g$  and  $\varepsilon_l$  are positive constants as before. The expected value is  $\bar{\varepsilon} = q\varepsilon_g - (1-q)\varepsilon_l$ .

Let  $R_g = aF + \varepsilon_g$ , and  $-R_l = -aF - \varepsilon_l$ , where  $a$  is a constant, and  $F$  as defined above. Expected profit function then becomes  $\pi^f = (2q-1)aF - c + \bar{\varepsilon}$ . Setting  $a = 1/(2q-1)$ ,  $2q \neq 1$ , we get  $\pi^f = F - c + \bar{\varepsilon}$ . Again, for suitable values of  $\varepsilon_g$  and  $\varepsilon_l$ , we can have  $\bar{\varepsilon} = 0$ , thus obtaining the profit function under certainty.

Therefore, both additive and multiplicative uncertainty can be obtained by imposing certain restrictions on gain and loss functions. Q.E.D

In general, additive and/or multiplicative uncertainty can be obtained from state-dependent uncertainty whenever the return function in one state is suitably represented as an *affine transformation* of the function in the other state. An affine transformation  $T$  of a variable  $x$  is  $T(x) = ax + b$ , where  $a$  and  $b$  are constants (Varian, 1992, p. 482). Obviously, this is a restrictive condition, which may or may not hold. Since state-dependent uncertainty is the general case, we expect it to be comparatively more relevant to reality.

### 3. SHARING WITH SYMMETRIC INFORMATION

Now we study how the entrepreneur would finance the project through external sources. External finance could be either in the form of debt or sharing. We start with sharing arrangement then move to debt. We assume that the financier is able to observe the state of the project, i.e., gain or loss, so the entrepreneur cannot misreport returns.

The entrepreneur acts as a *muḍārib*, i.e., does not contribute any portion of capital, and all capital belongs to the financier (*rabb-al māl*). Consequently, the term  $(1-q)R_l(e)$  drops from the entrepreneur's profit function. The entrepreneur gets a share  $\eta$  of gains in case of success, but bears no loss in case of failure. His profit function therefore, becomes:

$$\pi_s(e) = \eta q R_g(e) - c(e). \quad [4]$$

While the financier's profit function is:

$$v_s(e) = (1 - \eta)qR_g(e) - (1 - q)R_l(e). \quad [5]$$

The entrepreneur chooses effort to maximize  $\pi_s(e)$  subject to the constraint that  $v_s(e) \geq \bar{r}$ , where  $\bar{r}$  represents the opportunity cost for the financier. Assuming the constraint is not binding, first order condition is:

$$\eta q R'_g(e) = c'(e). \quad [6]$$

Comparing [6] with [3], we can achieve first-best level of effort if sharing ratio is set to

$$\begin{aligned} \eta^* &= \frac{\lambda(e^f)}{q} \\ &= 1 - \frac{(1-q)R'_l(e^f)}{qR'_g(e^f)} \end{aligned} \quad [7]$$

Consequently, the solution to the equation  $\eta^* q R'_g(e) = c'(e)$  is simply  $e^f$ . That is, when sharing ratio is chosen optimally, effort level can attain the same level of a self-financed entrepreneur. Obviously,  $\eta^*$  has to be a positive fraction.

PROPERTY 3:  $0 < \eta^* < 1$

This has already been shown in proving Property 1.

Next we need to show how optimizing the entrepreneur's profit function affects that of the financier. The following property demonstrates the result.

PROPERTY 4: If the constraint on the financier's profit function is not binding, then choosing the sharing ratio  $\eta^*$  maximizes the financier's profit function.

*Proof:* Let us ignore the constraint on the financier's profit function for while.

Differentiating  $v_s(e; \eta)$  with respect to  $e$  yields the first order condition for the financier:  $(1 - \eta)qR'_g - (1 - q)R'_l = 0$ .

Let the solution be  $e^s$ . Solving for  $\eta$ , we get  $\eta^* = 1 - (1 - q)R'_l(e^s) / qR'_g(e^s)$ . Compared with [7], this means that  $q\eta^* = \lambda(e^s)$ . By the uniqueness property of the profit function, this equality holds only at  $e^f$ . Consequently, we must have  $e^s = e^f$ . Q.E.D.

So as long as the financier's profit function is not less than the opportunity cost (so the constraint on  $v_s(e^f)$  is not binding), choosing optimal sharing ratio  $\eta^*$  maximizes profit functions of *both* the entrepreneur and the financier. This result points to the symmetry of payoff functions of the entrepreneur and the financier, and that both can be optimized simultaneously. To see this, note that  $\pi_f = \pi_s + v_s$ . Thus choosing the sharing ratio based on first-best maximizing conditions leads to maximizing both the financier's and the entrepreneur's profit functions.

### 3.1 PROPERTIES OF THE SHARING RATIO

It is insightful to examine characteristics of  $\eta^*$ . Recall that the entrepreneur's share is:

$$\eta^* = 1 - \frac{(1-q)R'_l(e^f)}{qR'_g(e^f)}$$

while that of the financier is

$$1 - \eta^* = \frac{(1-q)R'_l(e^f)}{qR'_g(e^f)}$$

Note that the financier's share is positively related to expected marginal loss. The greater the risk (or probability) of failure, or the greater the marginal loss, the greater the financier's share. This is intuitive as the financier is the one who bears the losses, so the rise in the financier's share reflects a compensation for the increase in risk he bears.

Note also that the entrepreneur's share is positively related to his marginal cost of effort. To see this, recall that the first order condition of the entrepreneur is  $\eta^*qR'_g = c'$ . Thus  $c'$  equals the nominator of  $\eta^*$ . Hence, the larger the marginal disutility of effort, the larger the entrepreneur's share in gains.

### 3.2 FINANCIER'S CONSTRAINT

Suppose the financier's constraint is binding, so that, at optimal sharing ratio  $\eta^*$ , we have  $v(e^f; \eta^*) < \bar{r}$ . Then  $\eta^*$  cannot be agreed upon, and a new sharing ratio, call it  $\hat{\eta}$ , is needed. Setting  $v_s(e^f; \hat{\eta}) = \bar{r}$  and solving for the new sharing ratio, we get:

$\hat{\eta} = (R_E - \bar{r})/qR_g$ , where  $R_E \equiv qR_g - (1-q)R_l$ . Note that as long as  $R_E > \bar{r}$ , then  $\hat{\eta}$  is a positive fraction. Now we compare  $\hat{\eta}$  with  $\eta^*$ .

PROPERTY 5:  $\eta^* \geq \hat{\eta}$ .

*Proof:* Define  $\bar{r}^* \equiv v_s(e^f; \eta^*)$ . When the opportunity cost is just equal to the the financier's unconstrained profit function, such that  $\bar{r} = \bar{r}^*$ , we would have  $\eta^* = \hat{\eta}$ , or, equivalently,  $1 - \eta^* = 1 - \hat{\eta}$ . Using the definitions of  $\eta^*$  and  $\hat{\eta}$ , we get:

$$\frac{(1-q)R_l'}{qR_g'} = \frac{(1-q)R_l + \bar{r}^*}{qR_g} .$$

This implies that  $R_l'/R_g' > R_l/R_g$  as long as  $\bar{r}^* > 0$ . Now suppose the opportunity cost rises by  $h \geq 0$ , so that  $\bar{r} = \bar{r}^* + h$ . In this case the constraint on the financier's profit function becomes binding, and hence:

$$\frac{(1-q)R_l'}{qR_g'} \leq \frac{(1-q)R_l + \bar{r}^*}{qR_g} + \frac{h}{qR_g}$$

Which means that  $(1 - \hat{\eta}) = (1 - \eta^*) + h/qR_g$ , and hence  $\eta^* \geq \hat{\eta}$ . Q.E.D.

In general, sharing ratio will be:

$$\eta = \min \{ \hat{\eta}, \eta^* \}. \quad [8]$$

That is, if the financier's constraint is binding, the sharing ratio will be  $\hat{\eta}$ ; otherwise it is  $\eta^*$ . This implies that the financier's profit function will be:  $v_s = \max \{ v_s(e^f; \eta^*), \bar{r} \}$ .

If we substitute for  $\hat{\eta}$  into the entrepreneur's profit function, we get:

$$\pi_s(e) = qR_g(e) - (1-q)R_l - c(e) - \bar{r}. \quad [9]$$

First order condition then is identical to that of a self-financed entrepreneur [2], so the solution is still  $e^f$ . This means that optimal effort level in a sharing contract is identical to that of first-best level, whether or not the financier's constraint is binding.

Nevertheless, it is important to note the difference between the case of binding and that of non-binding financier's constraint. Earlier studies (e.g. Hassan, 1985; Ahmed, 2002) simply assume the constraint is binding in equilibrium and thus solve for the sharing ratio accordingly. This is not a very interesting assumption in studying sharing

contracts. The classical argument against the efficiency of sharing obviously applies to the non-binding case. The contribution of the present model is that first-best effort can be obtained under sharing *even if* the financier's constraint is *not* binding. For this reason, discussion will focus on cases where the opportunity cost does not exceed the financier's optimal share in return. If it does, then the project probably is not worth undertaking. This also facilitates the analysis.

### 3.3 THE CASE OF CERTAINTY

It is useful to see how the model changes as probability of success approaches unity. Let us start from the optimal sharing ratio  $\eta^*$  with the financier's profit unconstrained. Differentiating  $v_s(e^f; \eta^*)$  with respect to  $q$ , we get:

$$\frac{\partial v_s(e^f; \eta^*)}{\partial q} = - \left( \frac{R'_l}{R'_g} R_g - R_l \right) < 0$$

The derivative is negative as long as  $v_s(e^f; \eta^*) > 0$ , which is true by assumption. Thus a rise in  $q$  would reduce the financier's payoff. At some point the financier's payoff becomes sufficiently small that the constraint becomes binding. The entrepreneur's profit function then will become as that in [9]. When  $q = 1$ , the term  $(1-q)R_l$  vanishes, and sharing becomes a 'fixed-payment' contract, whereby the financier gets  $\bar{r}$  while the entrepreneur gets  $\pi_s = R_g(e) - c(e) - \bar{r}$ .

This confirms the intuition that sharing arrangement is viable only under uncertainty. In case of certainty, sharing is irrelevant, as each party can determine his payoff without any loss of efficiency. This is consistent with the argument Muslim scholars provide for prohibiting a fixed compensation in a *muḍārabah* contract, independent of return. They argue that, since profits are uncertain, the fixed compensation might exceed realized profits, resulting in pure loss for the agent for the benefit of the principal. Consequently, payment in *muḍārabah* has to be on a sharing basis (Ibn Qudamah, 1997, 7:146). In absence of uncertainty, therefore, this reasoning ceases to hold, so that sharing is no more relevant, as predicted by our model. In reality, however, uncertainty is intrinsic in

every aspect of business activities, so an optimal ‘fixed-payment’ contract is purely hypothetical.

Finally, since  $R_g$  and  $R_l$  are concave, then as probability of success rises, optimal effort  $e^f$  also rises, and inefficiency resulting from uncertainty declines. Ultimately, when  $q = 1$ , optimal effort becomes equal to that in case of certainty.

### 3.4 PROPER SELECTION

So far we assumed that the entrepreneur carries out the project voluntarily. But what would make the entrepreneur commit himself to the project rather than, say, take the money and ‘run away’? The answer must rely on some form of balancing the costs and benefits of commitment and absconding with the money. If he absconds, the entrepreneur obviously loses his credibility and reputation as an entrepreneur. These costs could be reflected to a large extent in bankruptcy costs,  $b$ .

Suppose that if he absconds, the entrepreneur benefits  $D$ . Then the entrepreneur would commit to the project if  $D - b \leq R_E - c - \bar{r}^*$ . Suppose for simplicity that  $D \approx R_E$ . Thus, as long as  $b \geq c + \bar{r}^*$ , evaluated at optimal effort, the entrepreneur will prefer to carry out the project. Assuming the financier can estimate  $b$  with negligible costs, he would be able to select ‘true’ entrepreneurs upfront.

## 4. DEBT WITH SYMMETRIC INFORMATION

A ‘standard debt contract’ requires the entrepreneur to borrow capital at the start of the project, then repay the loan together with a fixed interest. The borrower does not provide guarantees for repayment, so it is not a secured debt. In case of success the borrower gets  $R_g(e) - i$ , where  $i$  is the required interest. In case of failure the borrower is forced to go bankrupt, costing him  $b$ . The borrower’s profit function therefore is:

$$\pi_d(e) = q(R_g(e) - i) - (1 - q)b - c(e). \quad [10]$$

The lender gets  $i$  in case of success, but loses  $R_l$  in case of failure. His profit function is:

$$v_d(e) = qi - (1 - q)R_l(e). \quad [11]$$

As before, the borrower maximizes [10] subject to the constraint  $v_d(e) \geq \bar{r}$ . The lender demands an interest equals the opportunity cost in addition to a risk premium. Charged interest will be such that  $v_d(e) = \bar{r}$ . Solving for  $i$  and substituting into the borrower's profit function, we get:

$$\pi_d(e) = qR_g(e) - (1-q)(R_l(e) + b) - c(e) - \bar{r}. \quad [12]$$

The borrower therefore maximizes [12] to obtain optimal level of effort.

It is apparent that first order condition for debt financing is identical to that of self-financing. This implies that debt achieves first-best level of efficiency. Consequently,  $e^d = e^f$ .

It is important to note that there is no reason why debt shall be selected in absence of informational asymmetry. Models that obtain optimality of debt do so only in case of asymmetric information. Nonetheless, the analysis in this section will make it much easier to examine the case of asymmetric information later in the paper.

#### 4.1 COST OF BANKRUPTCY

As in case of sharing, we look into the condition that makes the borrower commit himself to the project instead of 'running away' with the money and thus being in bankruptcy. Compared to sharing, we reach the following result.

PROPERTY 6: Other things equal, minimum cost of bankruptcy necessary to induce the entrepreneur to commit to the project under debt is larger than that under sharing. That is,  $b_d > b_s$ .

*Proof:* Net return from default is  $D - b$ . Without loss of generality, we assume as before that  $D \approx R_E$ . The entrepreneur prefers to commit rather than to default if  $\pi_d(e^f) \geq D - b$  under debt, and  $\pi_s(e^f) \geq D - b$  under sharing. To simplify the comparison, suppose that the opportunity cost is just equal to the unconstrained profit function of the financier, i.e.  $\bar{r} = \bar{r}^*$ . Solving for minimum bankruptcy cost for each case, we get  $b_s = c + \bar{r}^*$  and  $b_d = \frac{1}{q} b_s$ . Since  $\frac{1}{q} > 1$ , and  $b_d$  and  $b_s$  are positive, then  $b_d > b_s$ . Q.E.D

This result is not surprising. Under debt financing, ownership of capital is transferred from the lender to the borrower, in exchange for a promise to repay an identical amount plus interest. Given that the lender has no claim on capital returns, he needs to be assured of promised repayment. To make the promise credible, the borrower agrees to declare bankruptcy if he fails to repay. In other words, the lender gives up upside returns in exchange for being immune from downside losses through the borrower's promise to repay due debt. For the lender to accept this promise, bankruptcy costs must be sufficiently high. Under sharing, on the other hand, there is no specific promise the entrepreneur provides to the financier. The financier still owns the capital, so he enjoys upside returns and assumes downside losses. Consequently, bankruptcy costs are expected to be higher under debt than sharing.

The result points to an important difference between the two schemes. When bankruptcy costs are positively correlated with the size of the firm (as it appears to be the case; see Greenwald and Stiglitz, 1990, p. 162), it is more costly for large firms to default than it is for small firms. Thus debt tends to be provided to larger firms and excludes smaller ones. Sharing, on the other hand, is provided to a wider range of firms. Further, in an economy where debt financing is dominant, greater costs of bankruptcy are expected to prevail in order to protect lenders' interests. This indicates that, *ceteris paribus*, debt imposes greater transaction costs in a debt-dominated economy than sharing does in a sharing-dominated economy.

## 5. COMPARISONS

Now we compare debt and sharing in terms of expected profits. We first compare joint profits, then compare profits of each party under the two schemes.

### 5.1 JOINT PROFITS

PROPERTY 7: Expected joint profits under sharing exceed those under debt.

*Proof:* Joint profits in case of sharing are  $\pi^s + v^s = \pi^f$ , while those of debt are  $\pi^d + v^d = \pi^f - (1-q)b$ . Since  $(1-q)b > 0$ , then  $\pi^d + v^d < \pi^s + v^s$ . Q.E.D.

The reason joint profits are smaller for debt is positive bankruptcy costs. This is one way the Modigliani and Miller (1958) theorem on equivalence of different forms of finance is violated. As discussed above, bankruptcy costs are needed to induce the entrepreneur to commit to the project. The presence of such costs therefore helps avoid the problem of ‘lemons,’ i.e. entrepreneurs who are better off to run away with the financing. While bankruptcy costs have no impact on sharing, they impose dead-weight loss on debt financing.

## 5.2 ONE-TO-ONE COMPARISON

We now compare each agent’s profit holding the other constant. Suppose that the opportunity cost  $\bar{r}$  is at a level equal to expected profit of the financier in case of sharing, without imposing any restriction on the sharing ratio. Call that level of the opportunity cost  $\bar{r}^*$ . Then we have  $v_s(e^f; \eta^*) = \bar{r}^*$ . At this level we have  $\eta^* = \hat{\eta}$ , as before. If we substitute for  $\hat{\eta}$  into the profit function of the entrepreneur, we get

$$\pi_s(e^f; \bar{r}^*) = qR_g - (1-q)R_l - c - \bar{r}^*.$$

Under debt financing, the lender’s profit is equal to opportunity cost, so we have  $v_d = \bar{r}^*$ , and the borrower’s profit function is  $\pi_d(e^f; \bar{r}^*) = qR_g - (1-q)(R_l + b) - c - \bar{r}^*$ . As profits of the financier and the lender are equal, we are able to compare expected profits for the entrepreneur under the two contracts. The difference between the two is

$$\pi_s(e^f; \bar{r}^*) - \pi_d(e^f; \bar{r}^*) = (1-q)b > 0.$$

That is, holding profits of the financier and the lender equal, sharing generates higher expected profits for the entrepreneur than debt. This confirms the earlier result on the joint profits of the two contracts.

Now suppose that the opportunity cost is reduced by an amount  $h$ , so that  $\bar{r} = \bar{r}^* - h$ . As discussed earlier,  $h$  is bounded such that  $0 \leq h \leq \bar{r}^*$ . Then we can state the following result.

**PROPERTY 8:** Within the bounds of  $h$ , for  $h \leq (1-q)b$ , sharing Pareto-dominates debt. However, there is no range of  $h$  for which debt Pareto-dominates sharing.

*Proof:* Let us first summarize the payoff functions for each party.

The financier's payoff is  $v_s = \max\{\bar{r}^*, \bar{r}^* - h\}$  under sharing, and  $v_d = \bar{r}^* - h$  under debt.

The entrepreneur's payoff can be written as  $\pi_s = R_E - c - v_s$  under sharing, and as  $\pi_d = R_E - (1 - q)b - c - v_d$  under debt.

Note that:

$$v_s - v_d = \max\{h, 0\}, \text{ and} \quad [13]$$

$$\pi_s - \pi_d = (1 - q)b - (v_s - v_d). \quad [14]$$

The proof needs to address all ranges of  $h$ .

1. When  $h > 0$ , we have  $\bar{r}^* > \bar{r}^* - h$ , and  $v_s - v_d = h > 0$ . This implies that  $\pi_s - \pi_d = (1 - q)b - h$ . This is positive as long as  $h < (1 - q)b$ . So for this range of  $h$ , both the financier and the entrepreneur are better off signing a sharing contract instead of debt.

For  $h = (1 - q)b$  the payoffs for the entrepreneur under the two contracts will be identical, so  $\pi_s - \pi_d = 0$ . For  $h > (1 - q)b$ , the entrepreneur will be better off under debt instead of sharing. The financier however is better off under sharing since  $v_s - v_d > 0$ .

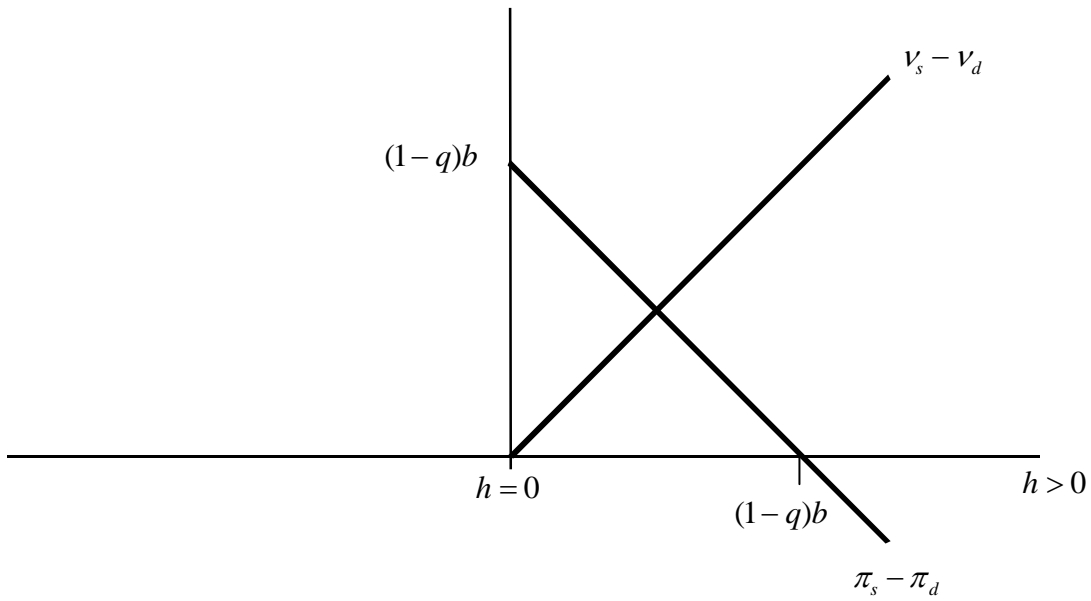
2. For  $h = 0$ , we have  $v_s = v_d$ , so the financier and the lender are indifferent between debt and sharing. For the entrepreneur,  $\pi_s - \pi_d = (1 - q)b$ , so the entrepreneur is better off under sharing.

It is apparent therefore, that for  $0 < h < (1 - q)b$  sharing strongly Pareto-dominates debt, but weakly so for  $h = 0$  and for  $h = (1 - q)b$ . However, debt never dominates sharing, neither strongly nor weakly. Q.E.D.

The following table and figure summarize these results.

**Table 1:** Gain in payoffs by signing a sharing contract instead of debt given an opportunity cost  $\bar{r} = \bar{r}^* - h$

$h$	To Financier ( $v_s - v_d$ )	To Entrepreneur ( $\pi_s - \pi_d$ )	Dominance of sharing v. Debt
$h = 0$	0	$+(1 - q)b$	weak
$0 < h < (1 - q)b$	$+h$	$+\left((1 - q)b - h\right)$	strong
$h = (1 - q)b$	$+h$	0	weak
$h > (1 - q)b$	$+h$	$-(h - (1 - q)b)$	none



**Figure 1:** Payoffs of sharing and debt contracts

## 6. ASYMMETRIC INFORMATION

Now we examine the model under asymmetric information. We assume that it is costly to observe effort level as well as the state of the world at the end of the period. We start with sharing, then move to debt.

### 6.1 SHARING

Under sharing, the financier relies on the entrepreneur to know realized return, and thus his share of profits. Although there are only two possible states of the world, effort might deviate from the optimal level and hence reported return might not be limited to either  $R_g(e^f)$  or  $R_l(e^f)$ .

To make the model more realistic, we assume that effort is subject to an uncontrollable multiplicative error, such that  $\tilde{e} = \tilde{\theta}e$ , where  $E\tilde{\theta} = 1$ . For example the entrepreneur might fall sick, or be misled or interfered by others such that he fails to implement optimal effort. Since  $E\tilde{\theta} = 1$ , however, the expected value of  $\tilde{e}$  is the same as certain effort,  $E\tilde{e} = e$ . As long as analysis is performed using expected effort, all previous results remain unchanged. This assumption allows for unintended sub-optimal effort by the entrepreneur without being punished by the financier if he finds out about it. As it is known in Islamic jurisprudence, a *muḍārib* is not held liable for uncontrollable failures. It should be noted though that since bankruptcy costs exceed disutility of effort, it is in the best interest of the entrepreneur to carry out the project at optimal effort, thus maximizing his return. Any deviation from optimal effort, therefore, is attributed to random error. But the financier cannot tell what was the effort level except at a cost.

If the entrepreneur announces  $R_g(e^f)$ , then there is no need for the financier to inspect or audit, since this is the maximum possible return. Any announced return below  $R_g(e^f)$  however could be due to either uncontrollable error or to misreporting by the entrepreneur. We assume that the financier adopts a random audit strategy, as this has been shown to be superior to deterministic audit (Townsend, 1979, p. 275-278; Khan, 1985, pp. 58-64; Mookherjee and Png, 1989; Dowd, 1992, p. 676). Intuitively, randomness creates better incentives for the entrepreneur to perform optimal effort. As

Brandenburger and Nalebuff (1997) point out, ‘Unpredictability is the key to effectiveness.’ (p. 222.)

Deterministic auditing, as mentioned in the introduction, is an essential ingredient for the result of optimality of debt. Williamson (1987) admits that he was not able to prove that debt is still optimal if stochastic monitoring is allowed (p. 164, ftn 5). According to Freixas and Rochet (1997), ‘standard debt contracts can be dominated if the situation allows for stochastic auditing procedures’ (p. 97). This has been shown by Mookherjee and Png (1989) and Krasa and Villamil (1994), among others.

### ***Random Audit Strategy***

Let  $\hat{R}$  be announced return by the entrepreneur. Since the project might gain and might lose,  $\hat{R}$  could be positive or negative. Since maximum possible gain is  $R_g(e^f)$  and maximum possible loss is  $R_l(e^f)$ , then  $-R_l(e^f) \leq \hat{R} \leq R_g(e^f)$ . Accordingly,

1. If  $\hat{R} = R_g(e^f)$ , no audit is needed since this is the maximum possible return.
2. For any lower level of return, the financier would audit with probability  $w$ .
3. If the entrepreneur is audited and found truthful, he gets his share of actual return,  $\eta R_g$ .
4. If the entrepreneur is audited and found untruthful, he is punished by  $y$ , where  $y \geq (\eta/1 - \eta)R_l(e^f)$ . This restriction is needed to assure that  $w$  lies within the unit interval.
5. Auditing costs the financier a positive amount  $m$ . Obviously,  $m$  cannot exceed the financier’s share in gains, thus  $m \leq (1 - \eta)R_g$ .

Let actual return be  $R_g$ . If the entrepreneur is not truthful, by misreporting his payoff is  $R_g - (1 - \eta)\hat{R}$  for  $\hat{R} \geq 0$ , where all functions of effort are evaluated at  $e^f$ . If  $\hat{R} < 0$  his payoff is  $R_g - \hat{R}$ . By misreporting the financier loses  $(1 - \eta)(R_g - \hat{R})$  for  $\hat{R} \geq 0$ , and  $(1 - \eta)R_g - \hat{R}$  for  $\hat{R} < 0$ . If the entrepreneur is audited and found untruthful, his payoff is  $-y$ . If he reports truthfully then he always gets  $\eta R_g$ . We are now able to compute optimal probability of auditing.

PROPERTY 9: Optimal probability of auditing is:

$$w^* = \begin{cases} w_1^* = (1-\eta)K_1, & 0 \leq \hat{R} \leq R_g \\ w_2^* = (1-\eta)K_2, & -R_l \leq \hat{R} \leq 0 \end{cases}$$

Where:

$$K_1 \equiv (R_g - \hat{R}) / (R_g - (1-\eta)\hat{R} + y), \text{ and}$$

$$K_2 \equiv (R_g - (1-\eta)^{-1}\hat{R}) / (R_g - \hat{R} + y).$$

When each probability is applied within its respective range of  $\hat{R}$ , we have:

$$0 \leq w_1^* < w_2^* < 1 \text{ for } \hat{R} \neq 0, \text{ and } 0 < w_1^* = w_2^* < 1 \text{ for } \hat{R} = 0.$$

*Proof:* We consider each case of announced returns.

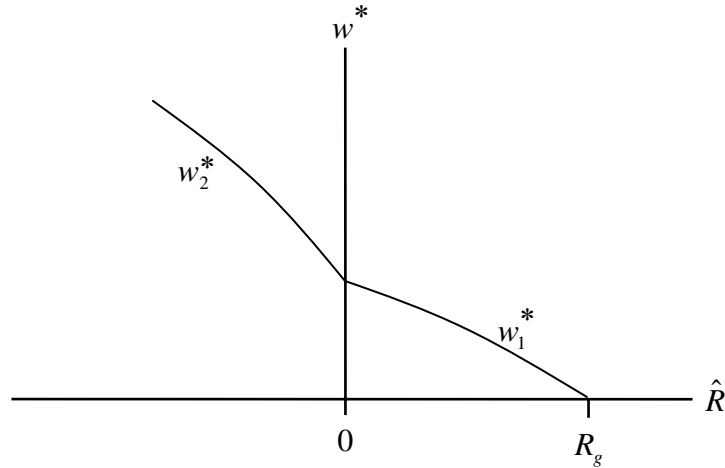
1.  $\hat{R} > 0$ : The entrepreneur will be truthful as long as his share of actual return is not less than expected gain from misreporting, i.e.  $\eta R_g \geq (1-w_1)(R_g - (1-\eta)\hat{R}) - w_1 y$ . Solving for  $w_1$ , we get  $w_1 \geq (1-\eta)K_1$ . Since the financier seeks to maximize return, he shall choose the smallest possible value of  $w_1$  to minimize expected costs of auditing. Consequently,  $w_1$  will be set equal to its lower bound, thus  $w_1^* = (1-\eta)K_1$ . Note that if announced return is  $R_g(e^f)$ , then  $w_1^* = 0$ , and no need for auditing in this case, as pointed out earlier.

Next we need to show that  $w_1^*$  is between zero and one. The entrepreneur cannot announce a return greater than  $R_g(e^f)$ , unless he wants to pay from his own resources; a possibility excluded from this model. So  $R_g - \hat{R} \geq 0$ , and  $K_1 \geq 0$ . Since  $R_g - (1-\eta)\hat{R} + y > R_g - \hat{R}$ , then  $0 \leq K_1 < 1$ . Since this fraction is multiplied by another fraction,  $1-\eta$ , it follows that  $0 \leq w_1^* < 1$ .

2.  $\hat{R} < 0$ : The entrepreneur will be truthful when  $\eta R_g \geq (1-w_2)(R_g - \hat{R}) - w_2 y$ . Thus the lower bound of  $w_2$  is  $w_2^* \geq (1-\eta)K_2$ . Since the financier minimizes expected cost of auditing, this would hold as an equality. When  $\hat{R} < 0$  the nominator and denominator of  $K_2$  are both positive, so that  $K_2 > 0$ . Since  $y \geq (\eta/1-\eta)R_l$  by assumption, and  $R_l \geq -\hat{R}$ , this implies that the nominator of  $K_2$  cannot be greater than its denominator. Thus  $0 < K_2 \leq 1$ . Since  $0 < 1-\eta < 1$ , it follows that  $0 < w_2^* < 1$ .

3.  $\hat{R} = 0$ : At this value of announced return,  $K_1 = K_2$ , so that  $w_1^* = w_2^*$ .

4. To show the relationship between the two probabilities, note first that each is decreasing in  $\hat{R}$ , as will be shown below. Let us start at  $\hat{R} = 0$ , where  $w_1^* = w_2^*$ , and allow  $\hat{R}$  to rise. Since  $\hat{R}$  becomes positive, only  $w_1^*$  will change. Since  $\partial w_1^* / \partial \hat{R} < 0$ ,  $w_1^*$  will decline. That is, as  $\hat{R}$  becomes positive,  $w_1^*$  becomes smaller than  $w_2^*$ . Now suppose that, starting from zero,  $\hat{R}$  declines, so that only  $w_2^*$  changes. Since  $\partial w_2^* / \partial \hat{R} < 0$ , then  $w_2^*$  will rise. So as  $\hat{R}$  becomes negative,  $w_2^*$  becomes larger than  $w_1^*$ . Consequently, with each probability used for the relevant range of  $\hat{R}$ ,  $w_1^* < w_2^*$ . (See Figure 2.) This completes the proof. Q.E.D.



**Figure 2:** Relationship of announced return and the probability of auditing

**Properties of  $w^*$ :**

1.  $w^*$  is decreasing in penalty cost,  $y$ . Higher penalty costs imply lower probability of auditing.

2. It is also decreasing in sharing ratio:

$$\partial w_1^* / \partial \eta = -(R_g - \hat{R})(R_g + y) / k_1^2 < 0, \text{ for } \hat{R} < R_g,$$

where  $k_1$  is the denominator of  $K_1$ .

Similarly,

$$\partial w_2^* / \partial \eta = -R_g / k_2 < 0,$$

where  $k_2$  is the denominator of  $K_2$ . The larger the entrepreneur's share in gains, the larger his payoff, and thus the less likely he may misreport returns. Conversely, the smaller his share, the more likely he may misreport in order to enlarge his payoff.

3. Also note that the greater the likelihood of success,  $q$ , the smaller is  $w^*$ . This is true since  $\eta^*$  is increasing in  $q$ , while  $w^*$  is decreasing in  $\eta^*$ . As probability of failure becomes larger, the financier needs to be more willing to audit.

4. As announced return rises, the financier becomes less willing to audit:

$$\partial w_1^* / \partial \hat{R} = -(1 - \eta)(\eta R_g + y) / k_1^2 < 0, \text{ and}$$

$$\partial w_2^* / \partial \hat{R} = -(\eta R_g + y) / k_2^2 < 0.$$

This property appears intuitive, since the larger the announced return, the smaller the magnitude of unreported gain, if any. When  $\hat{R} = R_g(e^f)$ , no auditing takes place. On the other hand, the smaller the announced return, the more likely auditing will take place. When  $\hat{R}$  is negative, not only  $w^*$  becomes larger, but it rises at an increasing rate, since it could be verified that  $\partial w_1^* / \partial \hat{R} < \partial w_2^* / \partial \hat{R}$ . This appears intuitive, as the financier becomes more worried when losses are announced.

Assuming the formula of optimal auditing is known to both parties, the entrepreneur can predict *a priori* what would the probability of audit be if he decides to misreport returns. After announcement, the financier computes optimal auditing probability, and draws upon its implementation accordingly.

We assume that the financier bears the cost of auditing, as it is argued that this is necessary for optimality of random auditing (Dowd, 1994, p. 676, footnote 9). The financier's profit function now becomes

$$v_s = (1 - \eta)qR_g - (1 - q)R_l - w^* m \geq \bar{r}. \quad [15]$$

It can be verified that if the constraint on the financier's payoff is binding, then the entrepreneur's payoff shall be:

$$\pi_s(e) = qR_g(e) - (1 - q)R_l(e) - c(e) - (\bar{r} + w^* m). \quad [16]$$

It can be easily seen that the corresponding optimal effort is still  $e^f$ . (Note that  $w^*$  is evaluated at  $e^f$ , so it is treated as a constant rather than as a function of  $e$ .) If the constraint is not binding,  $\pi_s(e)$  will be as in case of symmetric information (eq. [4]), and optimal effort would also be  $e^f$ .

This means that by choosing an optimal probability of auditing, efficient effort can be achieved, and thus first-best solution is attained even under asymmetric information.

## 6.2 DEBT FINANCING

Under debt financing, the lender needs to audit only when the borrower announces low returns, or losses, such that he is unable to repay, in which case the borrower is forced to default.

As noted earlier, debt becomes optimal under asymmetric information and deterministic auditing. The intuition is simple: since auditing is costly, the lender prefers to minimize auditing. The lender therefore asks for a fixed repayment, thus no auditing is needed. To deter the borrower from falsely announcing inability to repay, non-repayment forces the borrower into bankruptcy. Since bankruptcy costs are sufficiently high, this assures that it is in his interest to repay.

If effort is deterministic, then non-repayment happens only in case of failure, which occurs with probability  $1-q$ . Since effort is subject to a multiplicative error, then probability of non-repayment exceeds  $1-q$ , because return might fall below required repayment even if the project succeeds.

To find out probability of default, note that in case of failure, all possible values of the multiplicative error lead to non-repayment. The sum of these probabilities is one, and thus, non-repayment occurs in this case with probability  $1-q$ . In case of success, only a subset of the possible values of the error results in low effort such that realized gain is not sufficient for repayment. Let the sum of probabilities of these instances be  $\varepsilon_r$ . Then probability of non-repayment in case of success is  $\varepsilon_r q$ . Total probability of non-repayment therefore is  $\varepsilon_r q + 1 - q$ . To simplify the analysis, we assume that  $\varepsilon_r q$  is sufficiently small that it can be ignored. This does not affect the results below. Accounting for the term  $\varepsilon_r q$  only strengthens them.

Given that probability of non-repayment is  $1-q$ , the lender needs to audit with probability  $1-q$ . Thus the lender's expected payoff is  $v_d = qi - (1-q)(R_l(e) + m) \geq \bar{r}$ . Assuming the constraint is binding, the lender's payoff becomes simply  $\bar{r}$ , while the borrower's payoff becomes:

$$\pi_d = qR_g(e) - (1-q)(R_l(e) + m + b) - c(e) - \bar{r}. \quad [17]$$

First order condition is identical to that in case of symmetric information, so debt also achieves first-best effort despite informational asymmetry. The additional cost of monitoring affects total profit but not marginal conditions.

Note that we have assumed that the lender provides finance only to a borrower whose bankruptcy costs are sufficiently high that he is better off to commit to the project. This means that the borrower has no incentive to falsely declare inability to repay. So why should the lender undertake costly auditing of the borrower?

The answer is that, given inability to repay, the borrower is better off taking away some capital instead of nothing. Since the borrower has already defaulted, why not claim that the loss was 50% of capital when in fact it was only 20%, thus keeping 30% for himself? This means that the lender has to audit to find out actual losses and recover his remaining capital.

## 7. COMPARISONS UNDER ASYMMETRIC INFORMATION

We now compare expected profits under the two schemes. First we start with joint profits, then examine one-to-one payoffs.

### 7.1 JOINT PROFITS

In case of sharing, we have  $\pi^s + v^s = \pi^f - w^*m$ . In case of debt, joint profits are  $\pi^d + v^d = \pi^f - (1-q)(b+m)$ . Thus,  $(\pi^s + v^s) - (\pi^d + v^d) = (1-q)b + (1-q-w^*)m$ . A sufficient condition for this difference to be positive is  $(1-q-w^*) > 0$ . We now show that this condition holds when the constraint on the financier's profit function is not binding, and thus  $\eta = \eta^*$ .

PROPERTY 10: At optimal sharing ratio  $\eta^*$ ,  $1-q > w^*$ .

*Proof:* The proof proceeds in two steps. First we show that  $\eta^* > q$ , then show that  $w^* < 1-\eta^*$ . As a result,  $w^* < 1-q$ .

1. First we show that optimal sharing ratio exceeds probability of success. Recall that  $1-\eta^* = (1-q)R'_i/qR'_g$ . Since expected marginal gain is greater than marginal loss by assumption, so that  $R'_i/qR'_g < 1$ , then  $1-\eta^* < (1-q)$ . It follows that  $\eta^* > q$ .

2. From Property 9, at optimal sharing ratio  $\eta^*$ ,  $w_i^* = (1-\eta^*)K_i$  for  $i = 1, 2$ . We already showed that  $0 < K_1 < 1$  and  $0 < K_2 \leq 1$ . It follows then that  $w^* \leq (1-\eta^*)$ . Since  $w^* \leq 1-\eta^* < 1-q$  then  $1-q - w^* > 0$ . Q.E.D.

This result shows that expected joint profits under sharing exceed that under debt, even after costs of monitoring are included. This is attributed to two factors. First, bankruptcy costs affect debt but not sharing. Second, the lender is more likely to audit than the financier, as  $1-q > w^*$ , so expected costs of auditing are higher under debt. Note that the gap in joint profits between sharing and debt is larger when information is asymmetric than when it is symmetric. The presence of auditing costs represents greater dead weight loss for debt financing but not for sharing. Thus, sharing achieves higher returns under economic imperfections than debt.

We have assumed that cost of auditing is identical under the two schemes. There are good reasons, however, to believe that it is less costly for an equity holder to monitor than for a lender. An owner has a direct interest in the project, so he has stronger incentives to audit. Assuming a lower cost of auditing under sharing only strengthens the above results.

We also treated bankruptcy costs as equal under the two contracts. However, as argued earlier, minimum bankruptcy costs under debt are higher than that under sharing. So many projects that could be financed by sharing will be denied capital under debt. Entrepreneurs carrying out these projects are always better off under sharing.

The assumption based on which  $w^* < 1-q$  is that expected marginal gain exceeds marginal loss, or  $qR'_g > R'_i$ . This implies that probability of success has to be greater than the ratio of marginal loss to marginal gain for sharing to dominate debt. Thus a lower value of  $q$  could make debt more profitable. This points to an important

difference between the two schemes. Since the financier bears the risk of failure under sharing, he certainly requires the project to be viable, and thus likelihood of success has to meet a minimum level. A lender, on the other hand, does not directly bear such risk, so he is less restrictive in accepting projects with smaller  $q$ . In other words, debt financing dominates sharing for less viable projects, but for borrowers with higher bankruptcy costs. This is quite consistent with reality where banks care more for size of the firm than for the profitability of the project financed. Equity providers, like venture capitalists, care more about the viability of the venture than for the size of the firm. Thus the claim that equity tends to support more risky projects is not accurate.

In presence of optimal auditing, equity finance can control for risk of fraud. Auditing also has a disciplinary role for the management of the project, which helps the entrepreneur seek optimal effort needed to maximize returns. Taking these factors into consideration, therefore, projects financed through sharing should be less risky than those financed by debt.

## 7.2 ONE-TO-ONE COMPARISON

Just as we did under symmetric information, we set the financier's expected profit to be equal under both schemes, then vary the opportunity cost by  $h$  to evaluate gains from signing a sharing contract instead of debt.

PROPERTY 11: Under asymmetric information, sharing Pareto-dominates debt for  $h \leq (1-q)b + (1-q-w^*)m$ . Within the defined range of  $h$ , debt does not Pareto-dominates sharing.

*Proof:* The payoff functions are:

$$\pi_s = R_E - c - w^*m - v_s$$

$$\pi_d = R_E - c - (1-q)(m+b) - v_d$$

$$v_s = \max\{\bar{r}^*, \bar{r}^* - h\}$$

$$v_d = \bar{r}^* - h, \text{ and}$$

$$v_s - v_d = \max\{h, 0\}$$

$$\pi_s - \pi_d = (1-q)b + (1-q-w^*)m - (v_s - v_d)$$

1. For  $0 < h < (1-q)b + (1-q-w^*)m$ ,  $v_s = \bar{r}^*$  while  $v_d = \bar{r}^* - h$ . Thus,  $v_s - v_d = h$ , and  $\pi_s - \pi_d = (1-q)b + (1-q-w^*)m - h$ . Hence both the financier and the entrepreneur are better off signing a sharing contract for this range of the opportunity cost.

2. For  $h = (1-q)b + (1-q-w^*)m$ ,  $v_s - v_d = h$  while  $\pi_s - \pi_d = 0$ . So sharing weakly dominates debt on this range. Similarly:

3. For  $h = 0$ ,  $v_s - v_d = 0$  and  $\pi_s - \pi_d = (1-q)b + (1-q-w^*)m$ .

4. For  $h > (1-q)b + (1-q-w^*)m$ ,  $\pi_s - \pi_d < 0$  and  $v_s - v_d = h$ , so neither contract dominates the other. Q.E.D.

Note that the range of strong dominance of sharing over debt has expanded under asymmetric information. Sharing appears more immune to inefficiency and market friction than debt.

## 8. EXTENSIONS

This section extends the model in two directions: First is to consider probability of success as a function of effort instead of being exogenously determined. In many instances effort can make success more likely, not just make profit higher in case of success. As we show below, optimal sharing ratio will be derived so that sharing achieves first-best level of effort. Debt, however, fails to achieve first-best solution in this case.

The second extension concerns bankruptcy costs. We allow these costs to be influenced also by effort, so that greater effort makes it more costly to terminate the project and default. Again, sharing achieves efficient effort level but debt doesn't. Since the objective is to compare efficiency rather than expected profits, we shall restrict ourselves to the case of symmetric information for simplicity.

### 8.1 ENDOGENOUS PROBABILITY

It is more realistic to view likelihood of success to be influenced by effort. Higher effort can make success more probable, while low effort makes failure more probable.

In this case effort has two different effects: one on probability of success, the other is on magnitude of gain. Following many models in the literature, we assume probability as a strictly concave, continuous, twice differentiable function of effort:

$$q = q(e), \quad q' > 0, \quad q'' < 0.$$

Self-financed entrepreneur therefore chooses effort to maximize

$$\pi_f(e; q(e)) = q(e)R_g(e) - (1 - q(e))R_l(e) - c(e).$$

First order condition is:

$$qR'_g + q'R_g - (1 - q)R'_l + q'R_l - c' = 0. \quad [18]$$

Let the solution be  $e_q^f$ . By rearrangement, we get:

$$\psi(q'R_g + qR'_g) = c', \text{ where} \quad [19]$$

$$\psi = 1 - \frac{q'R_l + (1 - q)R'_l}{q'R_g + qR'_g}$$

Note that when  $q' = 0$ ,  $\psi = \lambda/q = \eta^*$ . Now we show that  $\psi$  is between zero and one.

PROPERTY 12:  $0 < \psi < 1$

*Proof:* From [19],  $c' > 0$  implies that  $\psi > 0$ . Thus  $qR'_g + q'R_g > (1 - q)R'_l + q'R_l$ .

Since all terms are positive, then  $0 < 1 - \psi < 1$ . Hence  $0 < \psi < 1$ . Q.E.D

### ***Sharing***

Assuming the constraint on the financier's profit function is not binding, the entrepreneur's problem is to choose effort to maximize:

$$\pi_s(e; q(e)) = \eta q(e)R_g(e) - c(e).$$

First order condition is:

$$[20] \quad \eta(q'R_g + qR'_g) = c'.$$

Let the solution be  $e_q^s$ . Comparing [20] and [19], it is apparent that first-best effort could be achieved if sharing ratio is set to  $\eta = \psi$ , so that  $e_q^s = e_q^f$ . Entrepreneur's profit function therefore becomes:

$$\pi_s(e_q^f) = \psi q(e_q^f) R_g(e_q^f) - c(e_q^f), \quad [21]$$

while that of the financier is:

$$v_s(e_q^f) = (1 - \psi) q(e_q^f) R_g(e_q^f) - (1 - q(e_q^f)) R_l(e_q^f). \quad [22]$$

It can be easily verified that, as long as the constraint on the financier's profit function is not binding, choosing  $\eta = \psi$  also maximizes the financier's profits.

### **Debt**

As before, we assume that the lender's return is equal to opportunity cost. Thus,  $v_d = \bar{r}$ . The borrower's objective function is:

$$\pi_d(e; q(e)) = q(e) R_g(e) - (1 - q(e)) (R_l(e) + b) - c(e) - \bar{r}. \quad [23]$$

Maximizing [23] with respect to effort yields the first order condition:

$$q R'_g + q' R_g - (1 - q) R'_l + q' R_l - c' + q' b = 0. \quad [24]$$

Let the solution be  $e_q^d$ . Comparing [24] with the first order condition of the self-financed entrepreneur [18], we can state the following property.

**PROPERTY 13:** With endogenous probability of success, optimal effort under debt exceeds that of first-best solution. That is:  $e_q^d > e_q^f$ .

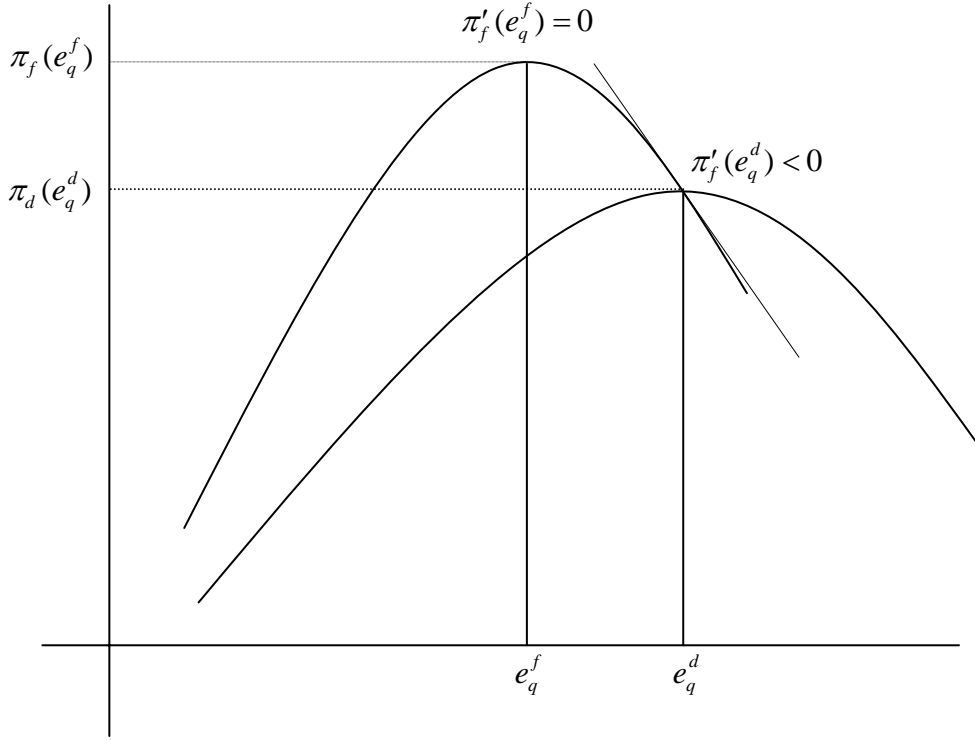
*Proof:* First order condition for the self-financed entrepreneur is stated in [18]. Compared to [24], we note that:

$$\underbrace{q R'_g + q' R_g - (1 - q) R'_l + q' R_l - c' + q' b}_\pi = 0$$

So the two are identical except for the last term  $q'b$ . The solution to [18] is  $e_q^f$ , at which  $\pi'_f(e_q^f) = 0$ . This means that at the solution of [24]  $e_q^d$ , we have  $\pi'_f(e_q^d) = -q'(e_q^d)b < 0$ . (See Figure 3.)

Since the profit function is positively sloped at values less than optimal effort, but negatively sloped afterwards, then  $e_q^d$  has to be larger than  $e_q^f$  to solve [24]. Hence  $e_q^d > e_q^f$ . Q.E.D

This result is consistent with earlier predictions in the literature. For example, Grossman and Hart (1982) argue that debt creates an incentive for managers to work harder to reduce probability of bankruptcy (cf. Harris and Raviv, 1991, p. 300). Unfortunately, harder does not always mean better. The borrower becomes under stress to avoid bankruptcy, leading to excess efforts and, thus, excess expenditures. Since investment in this model is associated with effort, this means that debt may lead to over-investment.



**Figure 3:** Profit functions under debt and sharing

### *Comparisons*

We start with joint profits. In case of sharing, we have:

$$\pi_s + v_s = \pi_f, \text{ evaluated at } e_q^f,$$

while in case of debt we have:

$$\pi_d + v_d = \pi_f - (1-q)b, \text{ evaluated at } e_q^d.$$

Note that  $\pi_f$  is evaluated at  $e_q^f$  in case of sharing, but at  $e_q^d$  in case of debt. The profit function  $\pi_f$  reaches its maximum at  $e_q^f$ , such that  $\pi_f(e_q^f) > \pi_f(e_q), \forall e_q \neq e_q^f$ . Consequently,  $\pi_f(e_q^f) > \pi_f(e_q^d)$ . Further, since  $(1-q)b > 0$ , it follows that joint profits in case of sharing exceed those in case of debt by  $\Delta + (1-q)b > 0$ , where  $\Delta = \pi_f(e_q^f) - \pi_f(e_q^d) > 0$ . There are now two sources of inefficiency in the debt contract: sub-optimal level of effort in addition to bankruptcy costs.

### ***One-to-One Comparison***

If we move to One-to-one comparison, we set  $v_s(e_q^f) = \bar{r}^*$ , as before, then let opportunity cost to vary by  $h$ . Following the same steps as before, we have (the subscript  $q$  is omitted for simplicity):

$$v_s(e^f) = \max\{\bar{r}^*, \bar{r}^* - h\} \text{ and } v_d = \bar{r}^* - h.$$

For the entrepreneur, we have:

$$\pi_s(e^f) = \pi_f(e^f) - v_s, \text{ where } \pi_f = qR_g - (1-q)R_l - c; \text{ and}$$

$$\pi_d(e^d) = \pi_f(e^d) - v_d.$$

For  $h > 0$ , we have

$$v_s - v_d = h > 0, \text{ and}$$

$$\pi_s(e^f) - \pi_d(e^d) = \Delta + (1 - q(e^d))b - h.$$

For  $h \leq 0$ , we have

$$v_s - v_d = 0, \text{ and}$$

$$\pi_s(e^f) - \pi_d(e^d) = \Delta + (1 - q(e^d))b - h.$$

Consequently, sharing contract Pareto-dominates debt for  $h \leq \Delta + (1 - q(e^d))b$ , with strong dominance for  $0 < h < \Delta + (1 - q(e^d))b$ . However, there is no range of  $h$  for which debt dominates sharing.

These results show that with endogenous probability, debt contract is inefficient. Knowing that his effort affects likelihood of success, the borrower becomes under pressure to achieve maximum gain, leading to excess effort,  $e_q^d > e_q^f$ . Yet for the relevant range of  $h$ , his expected gain is less than that under sharing. Sharing contract therefore economizes on cost of effort meanwhile allows for greater expected gain.

## **8.2 ENDOGENOUS BANKRUPTCY COSTS**

It is generally known that the entrepreneur, after spending time and effort for a certain project, prefers not to terminate the project, even if it were losing (Thaler, 1980; Arkes and Blumer, 1985). This “sunk-cost” phenomenon implies that costs of termination are larger the larger the invested effort. We can capture this effect by assuming bankruptcy cost to be an increasing function of effort instead of being

exogenously determined. Hence we assume  $b = b(e)$ , where  $b' > 0$ ,  $b'' < 0$ . To simplify the analysis, we assume that probability of success is exogenous.

In our model bankruptcy costs do not appear in the objective function of the self-financed entrepreneur, nor in that in case of sharing. We can simply carry out the results of the first section of the paper for this matter, where optimal effort is the same, i.e.  $e_b^f = e^f$ . Thus we focus on the case of debt to see the effects of this assumption.

### **Debt**

The borrower's profit function is:

$$\pi_d(e; b(e)) = qR_g(e) - (1-q)(R_l(e) + b(e)) - c(e) - \bar{r},$$

while that of the lender is:

$$v_d = \bar{r}.$$

First order condition for the borrower is:

$$qR'_g - (1-q)R'_l - c' - (1-q)b' = 0.$$

Let the solution be  $e_b^d$ . As before, this could be rewritten as  $\pi'_f(e_b^d) = (1-q)b'$ . Recall that at optimal effort  $e^f$ ,  $\pi'_f(e^f) = 0$ , and that it could be positive only for effort level smaller than  $e^f$ . Consequently, the following property holds.

PROPERTY 14: With endogenous cost of bankruptcy, effort level under debt is less than first-best solution; i.e.  $e_b^d < e^f$ .

The proof is similar to that of Property 13, but in the opposite direction.

### **Comparisons**

Joint profits under sharing are:

$$\pi_s(e_b^f) + v_s(e_b^f) = \pi_f(e^f).$$

In case of debt, we have:

$$\pi_d(e_b^d) + v_d(e_b^d) = \pi_f(e_b^d) - (1-q)b(e_b^d).$$

The difference between the two is  $\Delta + (1-q)b(e_b^d) > 0$ , where  $\Delta = \pi_f(e^f) - \pi_f(e_b^d) > 0$ .

For one-to-one comparison, we follow the same steps as before to obtain the range of  $h$  for which sharing contract Pareto-dominates debt  $h \leq \Delta + (1-q)b(e_b^d)$ .

These results point to the inefficiency of debt financing when bankruptcy costs are endogenous. The fear of rising costs of bankruptcy associated with high effort makes the borrower refrain from reaching optimal level of effort. This reduces overall expected profits, as well as his own profits for the relevant range of  $h$ .

### **Conclusion**

The fact that sharing arrangement is able to achieve first-best efficiency under a variety of conditions point to the flexibility and adaptability of sharing to different environments. Debt, in contrast, suffers from inefficient effort as well as dead-weight loss of bankruptcy costs.

Combining endogenous probability and endogenous bankruptcy costs would have conflicting impact on resulting effort. The gap in joint profits between sharing and debt, however, may not necessarily shrink. At best, the term  $\Delta$  might be zero, but the term  $(1-q)b$  would remain. In general, we expect bankruptcy costs to be determined by institutional factors than by individual characteristics. Hence we expect net effect on effort to be influenced by probability of success more than by bankruptcy costs.

## **9. CAPITAL SHARING: THE CASE OF *Musharakah***

Suppose the entrepreneur is willing to contribute part of the required capital of the project. What is the optimal share of capital he should contribute?

By contributing part of the capital, the entrepreneur becomes a partner as well as manager. In this structure, there will be two kinds of sharing ratios: One for sharing gains, while the other is for sharing capital. The latter is usually referred to in the literature as “cost” or “input sharing” (e.g. Braverman and Stiglitz, 1986). These two

ratios do not add to one, obviously, as they apply to two different variables. Since the entrepreneur contributes management, his share of gains should exceed his share in capital. We derive below optimal shares of gains and capital, and determine the relationship between them, as well as the sharing ratio in case of pure *mudharabah*. We assume for simplicity that information is symmetric and that the financier's profits are unconstrained.

Let  $\phi_1$  indicates the entrepreneur's share in gains, while  $\phi_2$  his share in capital. The payoff functions for the entrepreneur and the financier will be as follows:

$$\pi_m(e; \phi_1, \phi_2) = q\phi_1 R_g(e) - (1-q)\phi_2 R_l(e) - c(e) \quad [26]$$

$$v_m(e; \phi_1, \phi_2) = q(1-\phi_1)R_g(e) - (1-q)(1-\phi_2)R_l(e) \quad [27]$$

Where, as before,  $\pi_f = \pi_m + v_m$ . Now we show that the following property holds.

PROPERTY 15: At optimal sharing ratios of gain and capital,  $\phi_1^*$  and  $\phi_2^*$ , sharing contract achieves first-best effort level. Further:

$$\phi_1^* = \phi_2^* + \eta^*(1-\phi_2^*).$$

*Proof:* To achieve first-best efficiency, we start from first order condition of the entrepreneur in case of *mudharabah*, namely:  $q\eta^* R'_g = c'$ , the solution of which is  $e^f$ . Now suppose we add to the left-hand side of this first order condition the term  $xqR'_g$  and subtract  $\phi_2(1-q)R'_l$ ,  $\phi_2 > 0$ , such that the equation  $(\eta^* + x)qR'_g - \phi_2(1-q)R'_l = c'$  still holds at  $e^f$ . This requires that  $xqR'_g = \phi_2(1-q)R'_l$ , or, equivalently,  $x = \phi_2(1-\eta^*) > 0$ . By rearrangement, we get  $\phi_1^* = \phi_2^* + \eta^*(1-\phi_2^*)$ , which can be also written as  $\phi_1^* = \eta^* + \phi_2^*(1-\eta^*)$ . It can be easily verified that at these optimal shares, first order condition of the financier also holds at  $e^f$ . Q.E.D.

This result shows that optimal share in gains is greater than that in capital, consistent with the intuition stated earlier, but in contrast with the result of Allen and Lueck (1993). They argue that optimal shares in returns and input costs shall be equal. This can be true if we give up efficiency, so that effort level is below that of first-best solution. However, to induce the entrepreneur to supply efficient level of effort, his

share in gains has to exceed that in loss or capital. The following graph highlights this point.

In Figure 4, the line “equally shared marginal return” is a fraction of expected marginal return, so the two lines cannot intersect except at zero. Since the payoff of the entrepreneur in sharing is always less than that of a self financed entrepreneur, his effort level must be less, resulting in effort  $\hat{e}$ .

However, when shares are chosen optimally, marginal return in sharing becomes of a different slope than that of the self-financed, thus they may intersect at the optimal effort,  $e^f$ . Equal shares might be suitable when both the financier and the entrepreneur provide effort in managing the project (see below), where “effort” in our model denotes a broader concept than physical labor.

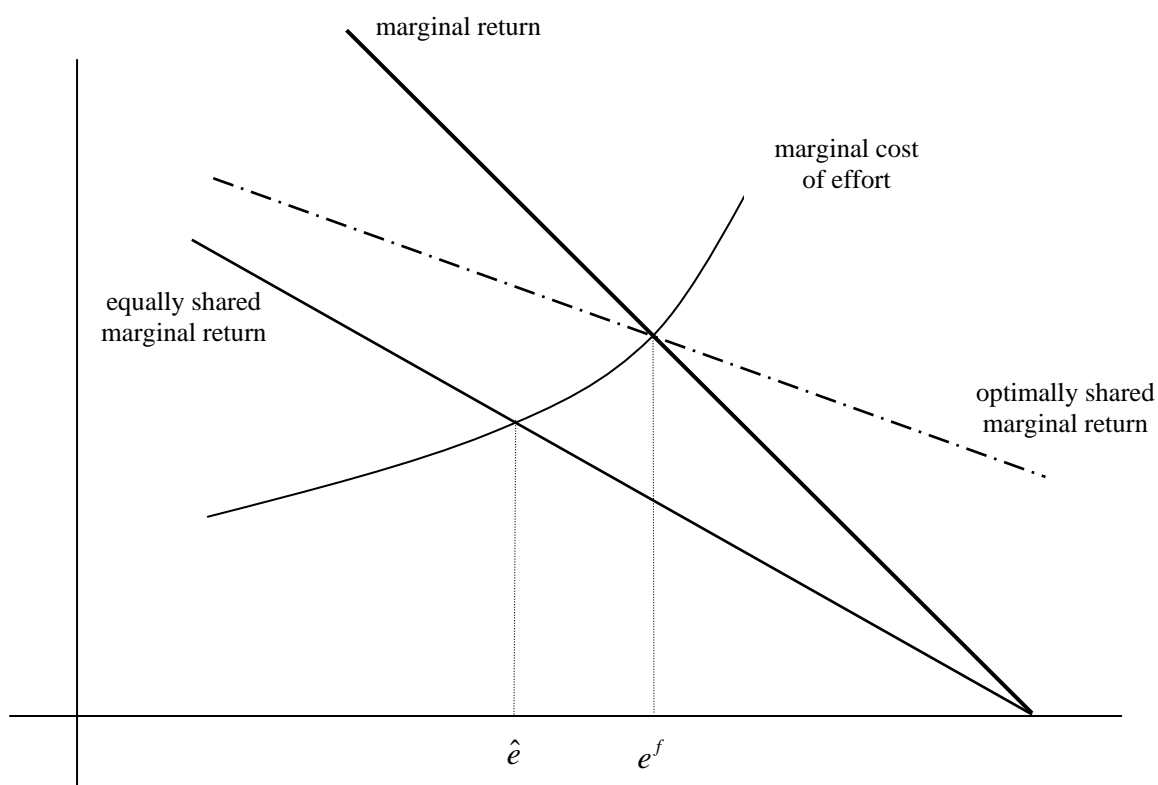
Note also that optimal share in gains in case of *musharakah* exceeds that in case of *mudharabah*, i.e.  $\phi_1^* > \eta^*$ . This leads to the following property:

PROPERTY 16: The entrepreneur reaps greater expected profits in case of *musharakah* than in case of *mudharabah*. The financier’s expected profits decline by the same magnitude.

*Proof:* The entrepreneur’s payoff in case of *mudharabah* is:  $\pi_s = q\eta^*R_g - c$ , while that in case of *musharakah* is  $\pi_m = q\phi_1^*R_g - (1-q)\phi_2^*R_l - c$ . Subtracting, we get

$$\pi_m - \pi_s = xqR_g - \phi_2^*(1-q)R_l.$$

This is positive as long as  $(1-\eta^*)qR_g > (1-q)R_l$ , which is true assuming the financier’s unconstrained profit function is positive, i.e.  $v_s(e^f; \eta^*) > 0$ . Thus,  $\pi_m - \pi_s > 0$ . Applying the same for the financier yields  $v_m - v_s = -(\pi_m - \pi_s)$ . Q.E.D



**Figure 4:** Marginal return and marginal cost of effort

### *Optimal Shares*

Since the financier's profits decline by sharing capital, *musharakah* could be implemented when the financier's return in case of *mudharabah* exceeds the opportunity cost by a margin not less than  $\pi_m - \pi_s$ . The entrepreneur therefore will decide whether or not to contribute to capital according to the financier's payoff compared to the opportunity cost. The following property summarizes the value of the two optimal shares.

PROPERTY 17: Given the opportunity cost  $\bar{r}$ , optimal shares of gain and capital are:

$$\phi_1^* = (v_s - (1 - \eta^*)\bar{r}) / v_s, \text{ and}$$

$$\phi_2^* = (v_s - \bar{r}) / v_s,$$

where  $v_s = q(1 - \eta^*)R_g(e^f) - (1 - q)R_l(e^f)$ .

*Proof:* The shares in gain and in capital are determined from the following two equations:

$$\pi'_m(e^f; \phi_1^*, \phi_2^*) = 0, \text{ and} \quad [28]$$

$$v_m(e^f; \phi_1^*, \phi_2^*) = \bar{r}. \quad [29]$$

Solving for  $\phi_1^*$  and  $\phi_2^*$ , and rearranging, we get the formulas stated above.

Alternatively, we can rewrite  $\phi_1^*$  as  $\phi_1^* = \eta^* + \phi_2^*(1 - \eta^*)$ , and substitute into  $\pi_m$  to get:  $\pi_m = \pi_s + \phi_2^* v_s$ . Since  $v_m - v_s = -(\pi_m - \pi_s)$ , this implies that  $v_m = (1 - \phi_2^*)v_s = \bar{r}$ . Thus  $1 - \phi_2^* = \bar{r} / v_s$ . Solving for  $\phi_1^*$  and  $\phi_2^*$ , we get the above formulas. Q.E.D.

Note that the larger the margin of the financier's return in case of *mudharabah*, the larger the share of capital that the entrepreneur should contribute in case of *musharakah*. On the other hand, if the opportunity cost is sufficiently high such that  $v_s(e^f; \eta^*) = \bar{r}$ , then  $\phi_2^* = 0$  and  $\phi_1^* = \eta^*$ , so that optimal *musharakah* in this case converges to a *mudharabah* contract. This is consistent with the Islamic jurisprudence view that *mudharabah* is a special form of *musharakah* (Ibn Qudamah, 1997, 7:120-121).

Comparing a *musharakah* contract to a debt contract now becomes straightforward. Since the lender's expected return is simply the opportunity cost, this means that the entrepreneur can achieve under *musharakah* a higher expected return than the borrower can get under debt financing. Given  $\pi_d$  in [12], the difference between the two is:

$$\begin{aligned} \pi_m - \pi_d &= (\phi_1 - 1)qR_g - (\phi_2 - 1)(1 - q)R_l + \bar{r} + (1 - q)b \\ &= (1 - q)b. \end{aligned}$$

In other words, the financier would get the same expected return under *musharakah* and debt, while the entrepreneur is strictly better off.

### ***Risk-Return Ratio Criterion***

Although the financier's expected payoff declines when the entrepreneur contributes capital, the risk-return ratio is still constant. Let us define the the risk-return ratio for the financier  $\rho$  as the ratio of expected loss to expected income, so that  $\rho_s = (1-q)R_l/v_s$  in case of *mudharabah*, and  $\rho_m = (1-\phi_2^*)(1-q)R_l/\bar{r}$  in case of *musharakah*. The following property then formulates this result.

PROPERTY 18: At optimal sharing ratios, risk-return ratio for the financier is the same under *musharakah* and *mudharabah*, and is lower than that under debt.

*Proof:* Since  $1-\phi_2^* = \bar{r}/v_s$ , then  $\rho_m = \rho_s$ . For the lender, we have  $\rho_d = (1-q)R_l/\bar{r}$ . Since the unconstrained profit function is not less than the opportunity cost, so that  $v_s \geq \bar{r}$ , then  $\rho_s \leq \rho_d$ . Q.E.D.

Judging by the risk-return ratio, therefore, the financier is worse off under debt compared with either *mudharabah* or *musharakah*.

### ***Diversification***

We next ask the following question concerning the entrepreneur: Is it more preferable for the entrepreneur to finance the entire project through his own resources, or should he invite outside investors through a *musharakah* arrangement?

Based on expected return, the entrepreneur is better off being self-financed, since  $\pi_f > \pi_m$ . But what about risk-return ratio? Let us define the risk return ratio for the self-financed entrepreneur as  $\sigma_f = (1-q)R_l/\pi_f$ , where  $\pi_f = qR_g - (1-q)R_l - c$ , and for the manager-partner as  $\sigma_m = \phi_2^*(1-q)R_l/\pi_m$ , where  $\pi_m = \phi_1^*qR_g - \phi_2^*(1-q)R_l - c$ . The following property compares these two parameters.

PROPERTY 19: Risk-return ratio for the entrepreneur under *musharakah* is smaller than that under self-financing, i.e.  $\sigma_m < \sigma_f$ .

*Proof:* Since  $\pi_m = \pi_s + \phi_2^* v_s$ , then we can write the risk-return ratio for the entrepreneur in case of *musharakah* as  $\sigma_m = (1-q)R_l / (v_s + \pi_s / \phi_2^*)$ . The risk return ratio under self-financing can also be written as  $\sigma_f = (1-q)R_l / (v_s + \pi_s)$ . Since  $0 < \phi_2^* < 1$ , then  $\pi_s / \phi_2^* > \pi_s$ , and thus  $\sigma_m < \sigma_f$ . Q.E.D.

This result shows that, in terms of risk-return ratio, the entrepreneur should invite outside investors to finance  $1 - \phi_2^*$  of the project instead of have it self-financed. This is consistent with the well known value of *diversification*, as the entrepreneur is better off not to put all capital into one project. The entrepreneur can invest  $\phi_2^*$  of capital in the project he manages, and contribute as a financier the remaining to other projects, such that the risk-return ratio for the resulting portfolio is lower than that of a single self-financed project.

Suppose he invests  $1 - \phi_2^*$  in a project with an expected return of  $\hat{r}$ , in addition to the project he is managing, assuming that each project requires the same amount of capital. Then the risk return ratio for the portfolio of the two projects becomes  $\zeta = (1-q)R_l / (\pi_s + \phi_2^* v_s + \hat{r})$ . As long as  $\hat{r} > (1 - \phi_2^*) v_s$ , the entrepreneur is better off diversifying, since  $\zeta < \sigma_f$ .

Finally, we examine the risk-return ratio of the borrower. Let us define risk-return ratio for the borrower as  $\sigma_d = (1-q)b / \pi_b$ , where  $\pi_b = qR_g - (1-q)(R_l + b) - c - \bar{r}$ . Comparing  $\sigma_b$  with  $\sigma_f$  and  $\sigma_m$ , we reach the following property.

PROPERTY 20: When bankruptcy cost is sufficiently large, risk-return ratio for the entrepreneur is larger under debt than under self-financing and, hence, than under *musharakah*. That is,  $\sigma_b > \sigma_f$  for  $b > \kappa R_l$ , where  $\kappa \equiv (\pi_f - \bar{r}) / (\pi_f + (1-q)R_l)$ .

*Proof:* Setting  $\sigma_b = \sigma_f$  and solving for  $b$ , we get  $b = \kappa R_l$ , where  $0 < \kappa < 1$ . Since  $\sigma_f > \sigma_m$ , then, for  $b > \kappa R_l$ , we have  $\sigma_b > \sigma_f > \sigma_m$ . Q.E.D.

Therefore, for a sufficiently large cost of bankruptcy, both the borrower and the lender are worse off under debt compared to sharing.

### **Partnership**

Now suppose the two parties contribute capital and management, so that it becomes a partnership or *sharikat 'inan*. How shall each party optimally select his share?

In a partnership, it is obvious that the two parties will not do the same thing at the same time. Rather, they will allocate responsibilities between them according to each party's proficiency and preferences. Consequently, each partner will be managing a different part of the project, while delegating the other partner to manage the remaining part. Assuming the structure of the project allows for such separability, we can view the project as consisting of two sub-projects. Each partner then will act as an entrepreneur on one sub-project, and as a financier for the other. This conforms to the Islamic view of partnerships, where each partner is considered as an agent for other (Ibn Qudamah, 1997, 7:128).

Suppose we have two partners,  $A$  and  $B$ . Partner  $A$  will be managing sub-project  $A$ , while partner  $B$  will be managing sub-project  $B$ . Let  $R_G(e_A, e_B)$  be the return in case of success of the whole project, as a function of effort of each partner, and  $R_L(e_A, e_B)$  be the return in case of failure. We need to show how optimal effort of each partner is determined.

Each partner will treat the gain function  $R_G$  as a function of his own effort, treating the effort of the other partner as absent, or zero. Thus his gain function becomes  $R_G(e_A | e_B = 0) > 0$ , and his loss function becomes  $R_L(e_A | e_B = 0) > 0$ . Similar reasoning applies to partner  $B$ . (For short, we write  $R_G(e_A | e_B = 0)$  as  $R_G(e_A | e_B)$ .) Accordingly, optimal effort  $e_i^*$ , for each sub-project can be obtained. We make the simplifying assumption that return functions of the whole project are additively separable in each sub-project:  $R_s(e_A^*, e_B^*) = R_s(e_A^* | e_B) + R_s(e_B^* | e_A)$ ,  $s = G, L$ .

Now let us define sharing ratios of each sub-project as follows:

$\omega_A$  : share of partner  $A$  in gains of sub-project  $A$ .

$\phi_A$  : share of partner  $A$  in capital (losses) of sub-project  $A$ .

$\omega_B$  : share of partner  $B$  in gains of sub-project  $B$ .

$\phi_B$  : share of partner  $B$  in capital (losses) of sub-project  $B$ .

Then, for partner  $i$ , the payoff function as an entrepreneur will be:

$$\pi_i = q\omega_i R_G(e_i|e_j) - (1-q)\phi_i R_L(e_i|e_j) - c(e_i),$$

and as a financier will be:

$$v_i = q(1-\omega_j)R_G(e_j|e_i) - (1-q)(1-\phi_j)R_L(e_j|e_i), \text{ where } i = A, B, i \neq j.$$

Each partner, acting as an entrepreneur, will maximize his payoff function for the relevant sub-project as if it were an independent project, given the other partner as a financier. Optimal values of the shares  $\omega_i$  and  $\phi_i$ ,  $i = A, B$ , are obtained through formulas presented in Property 17 above.

Let  $\Omega_A$  be the share of gains of the whole project for partner  $A$ , and  $\Omega_B$  be that of partner  $B$ . Obviously, we must have  $\Omega_A + \Omega_B = 1$ . Similarly, let  $\Phi_A$  be the share of capital (loss) for partner  $A$ , and  $\Phi_B$  be that of partner  $B$ . Again,  $\Phi_A + \Phi_B = 1$ .

The following property presents formulas for optimal values of sharing ratios for the whole project,  $\Omega_i$  and  $\Phi_i$ .

PROPERTY 21: Optimal partnership shares in gain are:

$$\Omega_A^* = \frac{\omega_A R_G(e_A^*(e_B)) + (1-\omega_B) R_G(e_B^*(e_A))}{R_G(e_A^*, e_B^*)}$$

$$\Omega_B^* = 1 - \Omega_A^*, \quad 0 < \Omega_i^* < 1, \quad i = A, B.$$

Optimal partnership shares in capital are:

$$\Phi_A^* = \frac{\phi_A R_G(e_A^*(e_B)) + (1-\phi_B) R_G(e_B^*(e_A))}{R_G(e_A^*, e_B^*)}$$

$$\Phi_B^* = 1 - \Phi_A^*, \quad 0 < \Phi_i^* < 1, \quad i = A, B.$$

*Proof:* Overall gain for partner  $A$  as an entrepreneur and as a financier is:

$$R_{G,A} = \omega_A R_G(e_A^*|e_B) + (1 - \omega_B) R_G(e_B^*|e_A),$$

while that for partner  $B$  is

$$R_{G,B} = \omega_B R_G(e_B^*|e_A) + (1 - \omega_A) R_G(e_A^*|e_B).$$

Overall gain for each partner must equal his share of gains of the whole project, so that  $\Omega_A R_G(e_A^*, e_B^*) = R_{G,A}$ , and  $\Omega_B R_G(e_A^*, e_B^*) = R_{G,B}$ . Solving, we get  $\Omega_A^*$  and  $\Omega_B^*$ . Since  $R_G(e_A^*, e_B^*) = R_G(e_A^*|e_B) + R_G(e_B^*|e_A)$ , then  $\Omega_B^* = 1 - \Omega_A^*$ . Since  $0 < \omega_i < 1$  then  $0 < \Omega_i^* < 1$ ,  $i = A, B$ .

Similarly, overall capital share for partner  $A$  is

$$R_{L,A} = \phi_A R_L(e_A^*|e_B) + (1 - \phi_B) R_L(e_B^*|e_A),$$

while that for partner  $B$  is

$$R_{L,B} = \phi_B R_L(e_B^*|e_A) + (1 - \phi_A) R_L(e_A^*|e_B).$$

Setting  $\Phi_A R_L(e_A^*, e_B^*) = R_{L,A}$  and  $\Phi_B R_L(e_A^*, e_B^*) = R_{L,B}$ , we get  $\Phi_A^*$  and  $\Phi_B^*$ . Since  $R_L(e_A^*, e_B^*) = R_L(e_A^*|e_B) + R_L(e_B^*|e_A)$ , then  $\Phi_B^* = 1 - \Phi_A^*$ . Since  $0 < \phi_i < 1$  then  $0 < \Phi_i^* < 1$ ,  $i = A, B$ . Q.E.D

This shows that the share in the whole project is a weighted average of the shares for each sub-project. Determining sharing ratio in a *mudharabah* arrangement paved the way for determining sharing ratios in a joint partnership, or *'inan* arrangement. Pure *mudharabah* therefore presents a “prototype” for a variety of forms of sharing, which allows for analyzing different sharing arrangements within a unified framework.

## 10. EVOLUTION OF SHARING

Based on previous sections, sharing Pareto-dominates debt financing for the relevant range of the opportunity cost. But is this enough for sharing to be widely adopted in reality? As game theory shows, a Pareto-optimal outcome may not prevail if there is an incentive for each party to exploit the other. In anticipation of that, debt may emerge as a response to non-cooperative behaviour. Unless there is a mechanism to reward cooperation and punish exploitation and defection, Pareto-inferior outcomes would prevail. In this section we suggest a setting through which these questions can be addressed and analyzed.

We assume an environment where there is no *a priori* commitment through binding contracts. Rather, selection of type of compensation evolves endogenously. Since there is no contractual commitment in this setting, there are no bankruptcy costs.

Prior to the start of the game, the financier advances capital to the entrepreneur, where the latter offers a form of security or collateral to the financier. Investment is performed during the period. By the end of the period, the entrepreneur is able separately to observe the state of the world, and thus announces the return. Afterwards, true returns become visible to both parties.

We assume that the entrepreneur does not enjoy announcing low or bad returns, as it negatively affects his image or reputation. Such announcement therefore costs him  $d$ . This cost is incurred even if returns are in fact low or negative. To simplify, we assume that the entrepreneur can announce either gains,  $R_g$ , or losses,  $R_l$ . No intermediate value may be announced.

If gains are announced, the financier always chooses to cooperate and thus share these gains based on optimal sharing ratio  $\eta^*$ . If losses are announced, the financier would either trust the entrepreneur, in which case he bears the entire loss of capital, or he might be suspicious, in which case he recovers his lost capital with interest from the security.

To make the game more realistic, we assume that the security is a durable asset (a vehicle or a real estate, say) that the entrepreneur uses for his personal needs. The financier then can personally utilize this asset. Utilization is made within the period, so

that the financier would owe the entrepreneur the ‘rent,’ assumed equivalent to the amount of loss and interest, which are determined *a priori*. In case of gain the due rent will be netted out from the financier’s share. In case of loss, the financier either pays the rent if he trusts the entrepreneur, or refuses to pay if he does not. The outcome of this ‘side exchange’ is simply to enable the financier to recover loss plus interest when announced loss is not trusted.

The following table summarizes the possible outcomes, where  $a_i$  represents the financier’s payoff, while  $b_i$  represents that of the entrepreneur.

		<i>Entrepreneur</i>	
		Cooperate	Defect
<i>Financier</i>	Cooperate	sharing $(a_1, b_1)$	‘share’ the downside $(a_3, b_3)$
	Defect	share the upside $(a_2, b_2)$	debt $(a_4, b_4)$

Each of the entrepreneur and the financier might be honest or dishonest. An honest entrepreneur chooses to cooperate, i.e., to announce actual returns. An honest financier would respond with cooperation, thus sharing gain in case of success, and assuming losses in case of failure. Thus the outcome (*Cooperate, Cooperate*) corresponds to a sharing contract.

A dishonest entrepreneur announces losses even in case of success. If the financier is honest and thus cooperates, the outcome (*Cooperate, Defect*) results in the entrepreneur exploiting the financier. The entrepreneur enjoys entire gains in case of success but let the financier assumes losses in case of failure. Thus the financier ‘shares’ only in losses.

If the financier suspects that the entrepreneur is dishonest, he would refuse to cooperate, and thus decides to recover announced loss plus interest from provided security, which results in the outcome (*Defect, Defect*). This corresponds to a standard debt contract.

If the financier is not honest, he would refuse to cooperate even if the entrepreneur honestly announces losses. The outcome thus is (*Defect, Cooperate*), whereby the financier exploits the entrepreneur. In this case the financier shares the gains but not the losses. In response, the entrepreneur would refuse to cooperate next round. Defection of the entrepreneur will not be a false announcement of losses; rather, he would simply refuse to announce returns whatsoever. Consequently, the financier has no choice but to avoid cooperation, resulting also in (*Defect, Defect*) outcome, corresponding to a debt contract.

Based on the specifications of gain and loss in the standard model, we can compute the payoffs for each party as follows:

Payoff	In Case of Success	In Case of Failure	Expected Payoff
$a_1$	$(1 - \eta^*)R_g$	$-R_l$	$v_s = \bar{r}^*$
$b_1$	$\eta^*R_g - c$	$-c - d$	$\pi_f - (1 - q)d - \bar{r}^*$
$a_2$	$(1 - \eta^*)R_g$	$\bar{r}$	$\bar{r}^* + (1 - q)(\bar{r} + R_l)$
$b_2$	$\eta^*R_g - c$	$-R_l - c - d - \bar{r}$	$\pi_f - (1 - q)(R_l + d + \bar{r}) - \bar{r}^*$
$a_3$	$-R_l$	$-R_l$	$-R_l$
$b_3$	$R_g + R_l - c - d$	$-c - d$	$\pi_f + R_l - d$
$a_4$	$\bar{r}$	$\bar{r}$	$\bar{r}$
$b_4$	$R_g - c - d - \bar{r}$	$-R_l - c - d - \bar{r}$	$\pi_f - d - \bar{r}$

Following the same approach of comparisons between payoffs under the two schemes, it can be shown that the financier and the entrepreneur are both better off under sharing when  $\bar{r} = \bar{r}^* - h$ ,  $0 < h < qd$ . We also assume that cost of announcing bad returns is bounded such that  $0 < d < (1 - q)R_l$ . We are now ready to state the following result.

PROPERTY 22: The above game between the entrepreneur and the financier is a Prisoner's Dilemma game.

*Proof:* The Prisoner's Dilemma game is defined as a game in which payoffs of the first (row) player are arranged as  $a_2 > a_1 > a_4 > a_3$ , and for the second (column) player as  $b_3 > b_1 > b_4 > b_2$  (e.g. Hirshleifer and Riley, 1992, p. 431). We now show that this is true for the above game.

Starting with the financier, we have  $a_2 - a_1 = (1 - q)(R_l + \bar{r}) > 0$ . Next, we have  $a_1 - a_4 = v_s - \bar{r} = h > 0$ , since  $\bar{r} = \bar{r}^* - h$ , and  $0 < h < qd$ . Finally,  $a_4 - a_3 = \bar{r} + R_l > 0$ .

For the entrepreneur, we have  $b_3 - b_1 = R_l - qd + \bar{r}^*$ . Since  $d < (1 - q)R_l$ , it follows that  $qd < R_l$ ; thus  $b_3 - b_1 > 0$ . Next, we have  $b_1 - b_4 = qd - h$ . This is positive since  $h < qd$  as assumed above. Finally,  $b_4 - b_2 = (1 - q)R_l - qd + \bar{r}^* - q\bar{r}$ , which is positive given that  $d < (1 - q)R_l$  and  $\bar{r}^* > \bar{r}$ . Q.E.D.

To give the game some feeling, let  $R_g = \sqrt{e}$ ,  $R_l = e^{0.7}$ ,  $c = 0.25 \exp(e)$ ,  $\bar{r} = .06$ ,  $d = .15$ , and  $q = .75$ . Then we have  $e^f = .895$ ,  $\eta^* = .772$ . The game payoffs are as follows:

		<i>Entrepreneur</i>	
		Cooperate	Defect
<i>Financier</i>	Cooperate	.093, .45	-.93, 1.35
	Defect	.34, .2	.06, .37

If the financier trusts the entrepreneur, the latter prefers to deceive the financier by announcing false returns or losses, thus gaining 1.35 instead of .45. If the financier predicts such choice, he would not trust the entrepreneur, and thus both will be at the bottom right corner of the game. Although both parties are better off at (*Cooperate, Cooperate*), applying a sharing arrangement, they end up at (*Defect, Defect*) using a debt contract.

Presenting the choice between debt and sharing in a Prisoner's Dilemma framework provides valuable insights into the problem, from both positive and normative perspectives. It shows that, from a normative point of view, sharing Pareto-dominates debt, as both parties are better off switching from  $(a_4, b_4)$  to  $(a_1, b_1)$ . However, on the positive side, debt prevails in presence of suspicion and lack of honesty, factors frequently cited in reality for avoiding sharing financing. Debt therefore emerges out of selecting dominant strategies, even though it is Pareto-inferior to sharing.

This inconsistency between Pareto-optimality and strategy dominance is a characteristic feature of the Prisoner's Dilemma game, as well as many real life problems. Many solutions that elicit cooperation in such games have been suggested in the literature. Here we shall only briefly outline some of them.

### **10.1 RECIPROCITY**

The most intuitive way to support mutual cooperation in a repeated Prisoner's Dilemma game is through reciprocity: A player will cooperate today expecting the other player to cooperate tomorrow. If he does not, he refuses to cooperate the day after, and so on. If the financier refuses to cooperate in bad states, the entrepreneur will refuse to cooperate in good states. So it is a reciprocal behaviour. Note that the entrepreneur in this model does not reciprocate dishonesty for dishonesty, as this is not morally acceptable from an Islamic perspective (Ali, 2001, p. 143). Rather, a dishonest move by the financier is reciprocated by refusing to announce any returns whatsoever by the entrepreneur.

From the entrepreneur's point of view, it could be viewed as a form of insurance. The financier insures the entrepreneur from capital losses in case of failure, in return for a share of gains in case of success. The 'premium' that the financier obtains is higher than the interest  $\bar{r}$  he obtains in case of debt. As an insurer, the financier will not accept to provide insurance unless the likelihood of success is sufficiently large to compensate for risk of loss. This is the same condition that we arrived at in the static model for sharing to Pareto-dominate debt.

Probably the most known experiments of reciprocal cooperation in Prisoner's Dilemma game are those implemented by Axelrod (1984; 1997), where 'tit-for-tat' strategy won the highest score against a variety of other strategies. In this strategy, a

player starts the game with cooperation, then does the same that his opponent does. Although the experiment was with a symmetric Prisoner's Dilemma game, the logic extends to asymmetric games (Axelrod, 1984, p. 17). Tit-for-tat was successful in eliciting cooperation in repeated interaction for an indefinite number of rounds.

A necessary condition for cooperation to emerge in a repeated play is to have a sufficiently high discount factor, and thus low discount rates. This is the well-known 'Folk theorem' in game theory (e.g. Kreps, 1990). Even if discount rates as such were high, the presence of *emotions* and moral sentiments help individuals balance short-term gains with long-term rewards, as Frank (1988) argues. If players have intrinsic *preferences* for reciprocity, then Fehr and Fischbacher (2002) argue that such players would cooperate if they are confident that the other players also cooperate, but defect otherwise. Consequently, a single-stage Prisoner's Dilemma game becomes a *coordination game*, in which both mutual cooperation and mutual defection are equilibrium outcomes. This is in contrast to a game played by selfish players, where the only equilibrium in a one-shot play is mutual defection.

In reality, experimental evidence suggests that people tend to cooperate more than to defect in a Prisoner's Dilemma settings (Roth, 1995). These experiments lend support to the view that individuals are not as selfish as standard economic theory presumes. Thus cooperation is more likely than economic theory usually predicts.

## 10.2 EVOLUTIONARY STABILITY

Instead of having a repeatedly played game between a financier and an entrepreneur, suppose we have a population of financiers and a population of entrepreneurs. At the beginning of each period, a member of each population is drawn to play with the other player. In this community, players who achieve the highest score survive and flourish, while those who score poorly diminish and decay. This is the principle of 'survival of the fittest' adopted in evolutionary game theory.

If players meet each other *randomly*, then the only evolutionary stable equilibrium is defection (e.g. Weibull, 1995). Consequently, tit-for-tat does not become a stable strategy in this environment. The reason is that a community playing tit-for-tat allows for a strategy like 'cooperate all the time' to flourish in the population. Such strategy does not have the ability to punish for defection, thus 'always cooperate' players could

be easily exploited by players who adopt the opposite strategy 'always defect.' Unless the proportion of 'always cooperate' is sufficiently small, the population will eventually be dominated by defectors (Hirshleifer and Riley, 1992, pp. 436-437). Modifications to tit-for-tat has been suggested to account for this deficiency (Nowak and Sigmund, 1993).

Now suppose that players meet each other in a somehow systematic manner, so that cooperative players meet each other more frequently than otherwise. This could take place if financiers can distinguish entrepreneurs who have played honestly in previous rounds. Similarly, entrepreneurs can distinguish financiers who have been cooperative in previous plays. If the tendency of cooperative agents to play with each other is sufficiently high, then cooperation could emerge as an evolutionary stable equilibrium, even if it is a dominated strategy otherwise (Hamilton, 1963; Gintis, 2000, pp. 266-271). The presence of 'correlated interaction' makes the outcome of evolutionary game theory part away from that of economic game theory (Skyrms, 1996, pp. 60-61). Cooperative players therefore cannot be easily exploited since they tend to play with each other more frequently than with defectors. Further, if they interact with each other sufficiently, players adopting tit-for-tat strategy can flourish even in a purely defective population (Axelrod, 1984, pp. 20-21).

Accordingly, cooperative agents playing with each other could drag the population into adopting cooperative strategies, resulting in Pareto-optimal outcomes in which every one is better off. 'Everyone would prefer being a cooperator in a society of cooperators to being a defector in a society of defectors' (Skyrms, 1996, p. 60). To promote sharing financing, therefore, honest agents should provide the example of cooperative behaviour to the rest of the population. Sharing then could emerge as an evolutionary stable equilibrium.

## 11. CONCLUSION

In presence of state-dependent uncertainty, marginal conditions deviate from those in case of certainty. Under such form of uncertainty, the classical argument of the inefficiency of sharing arrangements, be it equity financing or crop-sharing, ceases to hold. Sharing achieves first-best efficiency under a variety of conditions under which debt fails to do so. A reasonable explanation for the presence of debt when it is Pareto-dominated by sharing is the logic of the Prisoner's Dilemma game. Agents are better off under sharing, but each has an incentive to exploit the other, ending up with the inferior outcome of debt.

Many aspects of the adopted model as well as resulting properties appear intuitive, but break away from a large strand of the existing literature in fundamental dimensions. I hope this contributes to our understanding of the subject, and presents Islamic aspects of contracting in a more meaningful framework.

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